Work and Energy

Additional Practice E

Givens	*Solutions*
1. $m = 0.500$ g	$PE_i = KE_f$
$h = 0.250$ km	$mgh = KE_f$
$g = 9.81$ m/s^2	$KE_f = (0.500 \times 10^{-3}\text{ kg})(9.81\text{ m/s}^2)(0.250 \times 10^3\text{ m}) = \boxed{1.23\text{ J}}$
2. $d = 96.0$ m	**a.** $PE_1 = mgh = mgd(\sin\theta)$
$\theta = 18.4°$	$PE_2 = 0$ J
$m = 70.0$ kg	$ME_i = PE_1 + PE_2 = mgd(\sin\theta)$
$g = 9.81$ m/s^2	$ME_i = (70.0\text{ kg})(9.81\text{ m/s}^2)(96.0\text{ m})(\sin 18.4°) = \boxed{2.08 \times 10^4\text{ J}}$
	b. $PE_1 = 0$ J
	$PE_2 = mgh = mgd(\sin\theta)$
	$ME_f = PE_1 + PE_2 = mgd(\sin\theta) = ME_i$
	$ME_f = \boxed{2.08 \times 10^4\text{ J}}$
$v_1 = v_2 = 1.0$ m/s	**c.** $ME_i = ME_f$
$h_{1,f} = 20.0$ m	$PE_{1,i} + PE_{2,i} + KE_1 + KE_2 = PE_{1,f} + PE_{2,f} + KE_1 + KE_2$
	The kinetic energy of each passenger remains unchanged during the trip once the cars are in motion, so
	$PE_{1,i} + PE_{2,i} = PE_{1,f} + PE_{2,f}$
	$mgh_{1,i} + 0\text{ J} = mgh_{1,f} + PE_{2,f}$
	$PE_{2,f} = mgh_{1,f} - mgh_{1,f} = mgd(\sin\theta) - mgh_{1,f}$
	$PE_{2,f} = (70.0\text{ kg})(9.81\text{ m/s}^2)(96\text{ m})(\sin 18.4°) - (70.0\text{ kg})(9.81\text{ m/s}^2)(20.0\text{ m})$
	$PE_{2,f} = 2.1 \times 10^4\text{ J} - 1.37 \times 10^4\text{ J}$
	$PE_{2,f} = \boxed{7.0 \times 10^3\text{ J}}$

Givens | *Solutions*

3. $h_i = 75.0$ m

$v_i = 1.2$ m/s $+ 3.5$ m/s $= 4.7$ m/s

$v_f = 0$ m/s

$g = 9.81$ m/s^2

$m = 20.0$ g

$$PE_i + KE_i = PE_f + KE_f$$

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

$$h_f = \frac{\frac{1}{2}m(v_i^2 - v_f^2)}{mg} + h_i = \frac{v_i^2 - v_f^2}{2g} + h_i$$

$$h_f = \frac{(4.7 \text{ m/s})^2 - (0 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} + 75.0 \text{ m} = 1.1 \text{ m} + 75.0 \text{ m} = \boxed{76.1 \text{ m}}$$

---

4. $m = 25.0$ kg

$v = 12.5$ m/s

$g = 9.81$ m/s^2

$$PE_i = KE_f$$

$$mgh = \frac{1}{2}mv^2$$

$$h = \frac{v^2}{2g} = \frac{(12.5 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = \boxed{7.96 \text{ m}}$$

---

5. $m = 50.0$ g

$v_i = 3.00 \times 10^2$ m/s

$v_f = 89.0$ m/s

$$ME_i + \Delta ME = ME_f$$

$$ME_i = KE_i = \frac{1}{2}mv_i^2$$

$$ME_f = KE_f = \frac{1}{2}mv_f^2$$

$$\Delta ME = ME_f - ME_i = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$\Delta ME = \frac{1}{2}(50.0 \times 10^{-3} \text{ kg})[(89.0 \text{ m/s})^2 - (3.00 \times 10^2 \text{ m/s})^2]$$

$$\Delta ME = \frac{1}{2}(5.00 \times 10^{-2} \text{ kg})(7.92 \times 10^3 \text{ m}^2/\text{s}^2 - 9.00 \times 10^4 \text{ m}^2/\text{s}^2)$$

$$\Delta ME = \frac{1}{2}(5.00 \times 10^{-2} \text{ kg})(-8.21 \times 10^4 \text{ m}^2/\text{s}^2)$$

$$\Delta ME = \boxed{-2.05 \times 10^3 \text{ J}}$$

---

6. $m = 50.0$ g

$v_i = 3.00 \times 10^2$ m/s

$v_f = 89.0$ m/s

For upward flight,

$$PE_{1,f} - KE_{1,i} = \Delta ME_1$$

where

$$\Delta ME_1 = W_{net,1} = F_{net,1}\, h\,(\cos 180°) = (mg + F_{resist})\, h$$

For downward flight,

$$KE_{2,f} - PE_{2,i} = \Delta ME_2$$

Where

$$\Delta ME_2 = W_{net,2} = F_{net,2}\, h\,(\cos 0°) = (mg - F_{resist})\, h$$

Solving for h,

$$\Delta ME_2 - \Delta ME_1 = (mg - F_{resist})\, h - [-(mg + F_{resist})h] = 2\, mgh$$

$$h = \frac{\Delta ME_2 - \Delta ME_1}{2\, mg} = \frac{(KE_{2,f} - PE_{2,i}) - (PE_{1,f} - KE_{1,i})}{2\, mg}$$

$$KE_{2,f} = \frac{1}{2}mv_f^2$$

$$KE_{1,i} = \frac{1}{2}mv_i^2$$

$$PE_{1,f} = PE_{2,i} = mgh$$

$$h = \frac{\frac{1}{2}mv_f^2 - mgh - mgh + \frac{1}{2}mv_i^2}{2\, mg} = \frac{v_f^2 + v_i^2}{4g} - h$$

$$h = \frac{v_f^2 + v_i^2}{8g} = \frac{(89.0 \text{ m/s})^2 + (3.00 \times 10^2 \text{ m/s})^2}{(8)(9.81 \text{ m/s}^2)}$$

Givens	*Solutions*
	$h = \frac{7.92 \times 10^3\ \mathrm{m^2/s^2} + 9.00 \times 10^4\ \mathrm{m^2/s^2}}{(8)(9.81\ \mathrm{m/s^2})} = \frac{9.79 \times 10^3\ \mathrm{m^2/s^2}}{(8)(9.81\ \mathrm{m/s^2})}$ $h = \boxed{1.25 \times 10^3\ \mathrm{m} = 1.25\ \mathrm{km}}$
7. $m = 50.0$ kg $k = 3.4 \times 10^4$ N/m $x = 0.65$ m $h_f = 1.00\ \mathrm{m} - 0.65\ \mathrm{m} = 0.35$ m	$PE_{g,i} = PE_{elastic,f} + PE_{g,f}$ $mgh_i = \frac{1}{2}kx^2 + mgh_f$ $h_i = h_f + \frac{kx^2}{2\,mg} = 0.35\ \mathrm{m} + \frac{(3.4 \times 10^4\ \mathrm{N/m})(0.65\ \mathrm{m})^2}{(2)(50.0\ \mathrm{kg})(9.81\ \mathrm{m/s^2})} = 0.35\ \mathrm{m} + 15\ \mathrm{m}$ $h_i = \boxed{15\ \mathrm{m}}$
8. $h = 3.0$ m $g = 9.81\ \mathrm{m/s^2}$	$PE_i = KE_f$ $mgh = \frac{1}{2}mv_f^2$ $v_f = \sqrt{2gh} = \sqrt{(2)(9.81\ \mathrm{m/s^2})(3.0\ \mathrm{m})}$ $v_f = \boxed{7.7\ \mathrm{m/s}}$
9. $m = 100.0$ g $x = 30.0$ cm $k = 1250$ N/m	$PE_{elastic} = KE$ $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$ $v = \sqrt{\frac{kx^2}{m}} = \sqrt{\frac{(1250\ \mathrm{N/m})(30.0 \times 10^{-2}\ \mathrm{m})^2}{100.0 \times 10^{-3}\ \mathrm{kg}}}$ $v = \boxed{33.5\ \mathrm{m/s}}$
10. $m_w = 546$ kg $h = 5.64$ m $g = 9.81\ \mathrm{m/s^2}$ $m_{flyer} = 273$ kg	$PE_w = KE_{flyer}$ $m_w gh = \frac{1}{2}m_{flyer}v_{flyer}^2$ $v_{flyer} = \sqrt{\frac{2m_w gh}{m_{flyer}}} = \sqrt{\frac{(2)(546\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(5.64\ \mathrm{m})}{273\ \mathrm{kg}}}$ $v_{flyer} = \boxed{14.9\ \mathrm{m/s}}$

Work and Energy

Problem E

CONSERVATION OF MECHANICAL ENERGY

PROBLEM

A raindrop with a mass of 0.500 g falls to Earth from a height of 1.50 km. The raindrop reaches Earth's surface with a speed of 6.67 m/s. How much of the raindrop's mechanical energy is lost because of air resistance? Assume free-fall acceleration to be the same at 1.5 km above Earth's surface as it is at Earth's surface.

SOLUTION

1. **DEFINE**

Given: $m = 0.500$ g

$h = 1.50$ km

$v_f = 6.67$ m/s

$v_i = 0$ m/s

$g = 9.81$ m/s^2

Unknown: $\Delta ME = ?$

2. **PLAN** **Choose the equation(s) or situation:** Use the conservation of mechanical energy to account for energy dissipated through air resistance.

$$ME_i + \Delta ME = ME_f$$

The zero level for gravitational potential energy is the ground. Because the rain- drop starts at altitude h, the initial potential energy is its maximum value. Be- cause the raindrops' initial velocity is zero, the initial kinetic energy is zero.

$$ME_i = PE_i + KE_i = PE_i = mgh$$

When the raindrop reaches the ground, the gravitational potential energy is zero.

$$ME_f = PE_f + KE_f = KE_f = \frac{1}{2}mv_f^2$$

Substituting the last two equations into the first yields the following equation:

$$mgh + \Delta ME = \frac{1}{2}mv_f^2$$

Rearrange the equation(s) to isolate the unknown(s):

$$\Delta ME = \frac{1}{2}mv_f^2 - mgh$$

3. **CALCULATE** **Substitute the values into the equation(s) and solve:**

$$\Delta ME = \tfrac{1}{2}(0.500 \times 10^{-3}\ \text{kg})\ (6.67\ \text{m/s})^2$$
$$-(0.500 \times 10^{-3}\ \text{kg})\ (9.81\ \text{m/s}^2)\ (1.50 \times 10^3)$$
$$\Delta ME = 1.11 \times 10^{-2}\ \text{J} - 7.36\ \text{J}$$
$$\Delta ME = \boxed{-7.35\ \text{J}}$$

4. **EVALUATE** Most of the gravitational potential energy is given up through the interaction of the raindrop with the surrounding air. Only 0.15 percent of the initial energy remains as kinetic energy.

ADDITIONAL PRACTICE

1. What would be the kinetic energy of a 0.500 g raindrop if it fell 0.250 km without any resistance provided by air?
2. Angel's Flight, which has been called "the world's shortest railway," is a hill-climbing cable car, or *funicular*, that is located on Bunker Hill in downtown Los Angeles, California. The funicular consists of two small railway cars that are connected to each other by a cable. The cable is, in turn, wrapped around a large pulley that is attached to a motor. As one car rises up the hill, the other car descends to the street below.
   a. The tracks of Angel's Flight extend 96.0 m along the side of the hill at an angle of 18.4° with respect to the horizontal. If each car has a passenger with a mass of 70.0 kg, what is the total mechanical energy associated with the two passengers when the cars are about to leave the boarding platforms?
   b. What is the total mechanical energy associated with the two passengers when the cars arrive at their destinations?
   c. Except for a brief initial and final acceleration, the cars move in opposite directions at a constant speed of about 1.0 m/s. Suppose the ascending car is somewhere between street level and the midlevel. If the descending car is 20.0 m above street level, what is the gravitational potential energy associated with the passenger in the ascending car?
3. A toy rocket is at a height of 75.0 m and is moving upward with a speed of 1.2 m/s when it ejects a payload with a mass of 20.0 g. The payload has an initial upward speed relative to the rocket of 3.5 m/s. What is the height reached by the payload when its upward velocity is zero?
4. A 25.0 kg falling trunk strikes the ground with a speed of 12.5 m/s. Assuming that there is no loss of energy due to air resistance, what is the height from which the trunk falls?

5. If you were to neglect air resistance, a projectile fired straight up into the air would land again with the speed with which it was fired. (This is why it is dangerous to shoot a bullet directly upward.) However, some of the energy is lost because of air resistance. Suppose a 50.0 g projectile is fired upward with an initial speed of 3.00×10^2 m/s. If it lands with a speed of 89.0 m/s, how much mechanical energy is given up because of air resistance?
6. The air resistance that slows the projectile in problem 5 affects it both as it rises and falls. How high would the projectile rise if there were no air resistance? How high does it rise because of air resistance? (HINT: Use the work–kinetic energy theorem to describe the forces on the projectile as it goes up and again as it comes down. Then use these equations to solve for the projectile's maximum height. Use the data from problem 5 for the final calculation.)
7. A 50.0 kg circus performer jumps from a platform into a safety net below. The net, which has a force constant of 3.4×10^4 N/m, is stretched by 0.65 m. If the unstretched net is positioned 1.00 m above the ground, what is the height of the platform? Ignore the effects of air resistance.
8. A miniature golf course has a hole in which the fairway is 3.0 m above the green. If you hit the ball into the middle hole in a row of three, the ball will be directed to the green by a connecting pipe. Suppose the ball falls down most of the length of the pipe and slides the rest of the way without any loss of energy to friction. What is the ball's speed as it emerges from the pipe onto the green?
9. A 100.0 g arrow is pulled back 30.0 cm against a bowstring. If the spring constant of the bowstring is 1250 N/m, at what speed will the arrow leave the bow?
10. Because the wind speeds in Dayton, Ohio, are lower than at Kitty Hawk, North Carolina, the Wright brothers improved their launch mechanism for their 1904 flyer. Weights with a total mass of 546 kg were dropped 5.64 m from a derrick. These weights pulled a rope that was attached to the flyer, causing it to accelerate along a track for 5.64 m. This acceleration gave the flyer enough speed in addition to that provided by its propellers to take off. If the flyer, which had a mass of 273 kg, was initially at rest, what would its speed due to the falling masses be when the masses reached the ground? (Note that your answer is for a frictionless device; in reality, the speed of the flyer due to the weights was somewhat lower.)