Circular Motion and Gravitation

ADDITIONAL PRACTICE D

Givens | *Solutions*

1. $T = 88\ 643$ s

$m = 6.42 \times 10^{23}$ kg

$r_m = 3.40 \times 10^6$ m

$$r = \sqrt[3]{\frac{GmT^2}{4\pi^2}} = \sqrt[3]{\frac{\left(6.673 \times 10^{-11}\ \frac{\text{N•m}^2}{\text{kg}^2}\right)(6.42 \times 10^{23}\ \text{kg})(88\ 643\ \text{s})^2}{4\pi^2}}$$

$r = 2.04 \times 10^7$ m

$r_s = r - r_m = 2.04 \times 10^7\ \text{m} - 3.40 \times 10^6\ \text{m} = \boxed{1.70 \times 10^7\ \text{m}}$

2. $T = 5.51 \times 10^5$ s

$m = 1.25 \times 10^{22}$ kg

$$r = \sqrt[3]{\frac{GmT^2}{4\pi^2}} = \sqrt[3]{\frac{\left(6.673 \times 10^{-11}\ \frac{\text{N•m}^2}{\text{kg}^2}\right)(1.25 \times 10^{22}\ \text{kg})(5.51 \times 10^5\ \text{s})^2}{4\pi^2}}$$

$r = \boxed{1.86 \times 10^7\ \text{m}}$

3. $r = 4.22 \times 10^7$ m

$m = 5.97 \times 10^{24}$ kg

$$v_t = \sqrt{G\frac{m}{r}} = \sqrt{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)\frac{(5.97 \times 10^{24}\ \text{kg})}{(4.22 \times 10^7\ \text{m})}} = \boxed{3.07 \times 10^3\ \text{m/s}}$$

4. $r = 3.84 \times 10^8$ m

$m = 5.97 \times 10^{24}$ kg

$$v_t = \sqrt{G\frac{m}{r}} = \sqrt{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)\frac{(5.97 \times 10^{24}\ \text{kg})}{(3.84 \times 10^8\ \text{m})}} = \boxed{1.02 \times 10^3\ \text{m/s}}$$

5. $r = 3.84 \times 10^8$ m

$m = 5.97 \times 10^{24}$ kg

$$T = 2\text{p}\sqrt{\frac{r^3}{Gm}} = 2\text{p}\sqrt{\frac{(3.84 \times 10^8\ \text{m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(5.97 \times 10^{24}\ \text{kg})}} = 2.37 \times 10^6\ \text{s} = \boxed{27.4\ \text{d}}$$

6. $r = 4.50 \times 10^{12}$ m

$m = 1.99 \times 10^{30}$ kg

$$T = 2\text{p}\sqrt{\frac{r^3}{Gm}} = 2\text{p}\sqrt{\frac{(4.50 \times 10^{12}\ \text{m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\ \text{kg})}} = 5.20 \times 10^9\ \text{s} = \boxed{165\ \text{years}}$$

7. $r = 1.19 \times 10^6$ m

$T = 4.06 \times 10^5$ s

$$m = 4\text{p}^2\frac{r^3}{GT^2} = 4\text{p}^2\frac{(1.19 \times 10^6\ \text{m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(4.06 \times 10^5\ \text{s})^2} = \boxed{6.05 \times 10^{18}\ \text{kg}}$$

8. $r = 2.30 \times 10^{10}$ m

$T = 5.59 \times 10^5$ s

$m_s = 1.99 \times 10^{30}$ kg

$$m = 4\text{p}^2\frac{r^3}{GT^2} = 4\text{p}^2\frac{(2.30 \times 10^{10}\ \text{m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(5.59 \times 10^5\ \text{s})^2} = \boxed{2.30 \times 10^{31}\ \text{kg}}$$

$$\frac{m}{m_s} = \frac{2.30 \times 10^{31}\ \text{kg}}{1.99 \times 10^{30}\ \text{kg}} = \boxed{11.6}$$

Name:____________________________ Class:_________________ Date:_________________

Circular Motion and Gravitation

Problem D

PERIOD AND SPEED OF AN ORBITING OBJECT

PROBLEM

A satellite in geostationary orbit rotates at exactly the same rate as Earth, so the satellite always remains in the same position relative to Earth's surface. The period of Earth's rotation is 23 hours, 56 minutes, and 4 seconds. What is the altitude of a satellite in geostationary orbit?

SOLUTION

1. **DEFINE**

Given: T = 23:56:04 = 86 164 s

Unknown: r_s = ?

2. **PLAN** **Choose an equation or situation:** Use the equation for the period of an object in circular orbit, and rearrange the equation to solve for r.

$$T = 2\pi\sqrt{\frac{r^3}{Gm}}$$

$$T^2 = 4\pi^2 \frac{r^3}{Gm}$$

$$r^3 = \frac{GmT^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{GmT^2}{4\pi^2}}$$

Use **Table 1** in your textbook to find Earth's mass (m) and radius (r_e).

$r_e = 6.38 \times 10^6$ m (from Table 1)
$m = 5.97 \times 10^{24}$ kg (from Table 1)

Once you have found the total radius of the orbit (r), you can subtract Earth's radius (r_e) to find the altitude of the satellite (r_s).

$$r_s = r - r_e$$

3. **CALCULATE Substitute the values into the equations and solve:**

$$r = \sqrt[3]{\frac{\left(6.673\times10^{-11}\ \text{N}\bullet\frac{\text{m}^2}{\text{kg}^2}\right)(5.97\times10^{24}\ \text{kg})\ (86\ 164\ \text{s})^2}{4\pi^2}} = 4.22\times10^7\ \text{m}$$

$$r_s = r - r_e = (4.22\times10^7\ \text{m}) - (6.38\times10^6\ \text{m}) = 3.58\times10^7\ \text{m}$$

4. **EVALUATE** A satellite in geostationary orbit is at an altitude of 3.58×10^7 m = 35,800 km.

ADDITIONAL PRACTICE

1. The period of Mars' rotation is 24 hours, 37 minutes, and 23 seconds. At what altitude above Mars would a "Mars-stationary" satellite orbit?
2. Pluto's moon, Charon, has an orbital period of 153 hours. How far is Charon from Pluto?
3. The orbital radius of a satellite in geostationary orbit is 4.22×10^7 m (see sample problem on previous page). What is the orbital speed of a satellite in geostationary orbit?
4. Earth's moon orbits Earth at a mean distance of 3.84×10^8 m. What is the moon's orbital speed?
5. Earth's moon orbits Earth at a mean distance of 3.84×10^8 m. What is the moon's orbital period? Express your answer in Earth days.
6. Use data from **Table 1** in your textbook to calculate the length of Neptune's "year" (the period of its orbit around the Sun). Express your answer in Earth years.
7. The asteroid (45) Eugenia has a small moon named S/1998(45)1. The moon orbits Eugenia once every 4.7 days at a distance of 1.19×10^3 km. What is the mass of (45) Eugenia?
8. V404 Cygni is a dark object orbited by a star in the constellation Cygnus. Many astronomers believe the object is a black hole. Suppose the star's orbit has a mean radius of 2.30×10^{10} m and a period of 6.47 days. What is the mass of the black hole? How many times larger is the mass of the black hole than the mass of the sun?