Momentum and Collisions

ADDITIONAL PRACTICE E

Givens | *Solutions*

1. $m_1 = 155$ kg
$\mathbf{v_{1,i}} = 6.0$ m/s forward
$v_{2,i} = 0$ m/s
$\mathbf{v_f} = 2.2$ m/s forward

$$m_2 = \frac{m_1\mathbf{v_{1,i}} - m_1\mathbf{v_f}}{\mathbf{v_f} - \mathbf{v_{2,i}}} = \frac{(155\text{ kg})(6.0\text{ m/s}) - (155\text{ kg})(2.2\text{ m/s})}{2.2\text{ m/s} - 0\text{ m/s}}$$

$$m_2 = \frac{930\text{ kg•m/s} - 340\text{ kg•m/s}}{2.2\text{ m/s}} = \frac{590\text{ kg•m/s}}{2.2\text{ m/s}}$$

$$m_2 = \boxed{270\text{ kg}}$$

2. $v_{1,i} = 10.8$ m/s
$v_{2,i} = 0$ m/s
$v_f = 10.1$ m/s
$m_1 = 63.0$ kg

$$m_2 = \frac{m_1 v_{1,i} - m_1 v_f}{v_f - v_{2,i}} = \frac{(63.0\text{ kg})(10.8\text{ m/s}) - (63.0\text{ kg})(10.1\text{ m/s})}{10.1\text{ m/s} - 0\text{ m/s}}$$

$$m_2 = \frac{6.80 \times 10^2\text{ kg•m/s} - 6.36 \times 10^2\text{ kg•m/s}}{10.1\text{ m/s}} = \frac{44\text{ kg•m/s}}{10.1\text{ m/s}} = \boxed{4.4\text{ kg}}$$

3. $\mathbf{v_{1,i}} = 4.48$ m/s to the right
$\mathbf{v_{2,i}} = 0$ m/s
$\mathbf{v_f} = 4.00$ m/s to the right
$m_2 = 54$ kg

$$m_1 = \frac{(54\text{ kg})(4.00\text{ m/s}) - (54\text{ kg})(0\text{ m/s})}{4.48\text{ m/s} - 4.00\text{ m/s}} = \frac{(54\text{ kg})(4.00\text{ m/s})}{0.48\text{ m/s}}$$

$$m_1 = \boxed{450\text{ kg}}$$

4. $m_1 = 28 \times 10^3$ kg
$m_2 = 12 \times 10^3$ kg
$\mathbf{v_{1,i}} = 0$ m/s
$\mathbf{v_f} = 3.0$ m/s forward

$$\mathbf{v_{2,i}} = \frac{(m_1 + m_2)\mathbf{v_f} - m_1\mathbf{v_{1,i}}}{m_2}$$

$$\mathbf{v_{2,i}} = \frac{(28 \times 10^3\text{ kg} + 12 \times 10^3\text{ kg})(3.0\text{ m/s}) - (28 \times 10^3\text{ kg})(0\text{ m/s})}{12 \times 10^3\text{ kg}}$$

$$\mathbf{v_{2,i}} = \frac{(4.0 \times 10^4\text{ kg})(3.0\text{ m/s})}{12 \times 10^3\text{ kg}}$$

$$\mathbf{v_{2,i}} = \boxed{1.0 \times 10^1\text{ m/s forward}}$$

5. $m_1 = 227$ kg
$m_2 = 267$ kg
$\mathbf{v_{1,i}} = 4.00$ m/s to the left $= -4.00$ m/s
$\mathbf{v_f} = 0$ m/s

$$\mathbf{v_{2,i}} = \frac{(m_1 + m_2)\mathbf{v_f} - m_1\mathbf{v_{1,i}}}{m_2}$$

$$\mathbf{v_{2,i}} = \frac{(227\text{ kg} + 267\text{ kg})(0\text{ m/s}) - (227\text{ kg})(-4.00\text{ m/s})}{267\text{ kg}} = 3.40\text{ m/s}$$

$$\mathbf{v_{2,i}} = \boxed{3.40\text{ m/s to the right}}$$

Givens	*Solutions*
6. $m_1 = 9.50$ kg $\mathbf{v_{1,i}} = 24.0$ km/h to the north $m_2 = 32.0$ kg $\mathbf{v_f} = 11.0$ km/h to the north	$\mathbf{v_{2,i}} = \frac{(m_1 + m_2)\mathbf{v_f} - m_1\mathbf{v_{,1}}}{m_2}$ $\mathbf{v_{2,i}} = \frac{(9.5 \text{ kg} + 32.0 \text{ kg})(11.0 \text{ km/h}) - (9.50 \text{ kg})(24.0 \text{ km/h})}{32.0 \text{ kg}}$ $\mathbf{v_{2,i}} = \frac{(41.5 \text{ kg})(11.0 \text{ km/h}) - 228 \text{ kg•km/h}}{32.0 \text{ kg}} = \frac{456 \text{ kg•km/h} - 228 \text{ kg•km/h}}{32.0 \text{ kg}}$ $= \frac{228 \text{ kg•km/h}}{32.0 \text{ kg}}$ $\mathbf{v_{2,i}} = \boxed{7.12 \text{ km/h to the north}}$
7. $m_1 = m_2$ $v_{1,i} = 89$ km/h $v_{2,i} = 69$ km/h	Because $m_1 = m_2$, $v_f = \frac{v_{1,i} + v_{2,i}}{2} = \frac{89 \text{ km/h} + 69 \text{ km/h}}{2} = \frac{158 \text{ km/h}}{2} = 79 \text{ km/h}$ $v_f = \boxed{79 \text{ km/h}}$
8. $m_1 = 3.0 \times 10^3$ kg $m_2 = 2.5 \times 10^2$ kg $\mathbf{v_{2,i}} = 3.0$ m/s down $= -3.0$ m/s $\mathbf{v_{1,i}} = 1.0$ m/s up $= +1.0$ m/s	$\mathbf{v_f} = \frac{m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}}}{m_1 + m_2} = \frac{(3.0 \times 10^3 \text{ kg})(1.0 \text{ m/s}) + (2.5 \times 10^2 \text{ kg})(-3.0 \text{ m/s})}{(3.0 \times 10^3 \text{ kg}) + (2.5 \times 10^2 \text{ kg})}$ $\mathbf{v_f} = \frac{3.0 \times 10^3 \text{ kg•m/s} - 7.5 \times 10^2 \text{ kg•m/s}}{3.2 \times 10^3 \text{ kg}} = \frac{2.2 \times 10^3 \text{ kg•m/s}}{3.2 \times 10^3 \text{ kg}}$ $\mathbf{v_f} = 0.69 \text{ m/s} = \boxed{0.69 \text{ m/s upward}}$
9. $m_1 = (2.267 \times 10^3 \text{ kg}) + (5.00 \times 10^2 \text{ kg}) = 2.767 \times 10^3$ kg $m_2 = (1.800 \times 10^3 \text{ kg}) + (5.00 \times 10^2 \text{ kg}) = 2.300 \times 10^3$ kg $\mathbf{v_{1,i}} = 2.00$ m/s to the left $= -2.00$ m/s $\mathbf{v_{2,i}} = 1.40$ m/s to the right $= +1.40$ m/s	$\mathbf{v_f} = \frac{m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}}}{m_1 + m_2} = \frac{(2.767 \times 10^3 \text{ kg})(-2.00 \text{ m/s}) + (2.300 \times 10^3 \text{ kg})(1.40 \text{ m/s})}{2.767 \times 10^3 \text{ kg} + 2.300 \times 10^3 \text{ kg}}$ $\mathbf{v_f} = \frac{-5.53 \times 10^3 \text{ kg•m/s} + 3220 \text{ kg•m/s}}{5.067 \times 10^3 \text{ kg}} = \frac{-2310 \text{ kg•m/s}}{5067 \text{ kg}} = -0.456 \text{ m/s}$ $\mathbf{v_f} = \boxed{0.456 \text{ m/s to the left}}$

Problem E

PERFECTLY INELASTIC COLLISIONS

PROBLEM

The Chinese giant salamander is one of the largest of salamanders. Suppose a Chinese giant salamander chases a 5.00 kg carp with a velocity of 3.60 m/s to the right and the carp moves with a velocity of 2.20 m/s in the same direction (away from the salamander). If the speed of the salamander and carp immediately after the salamander catches the carp is 3.50 m/s to the right, what is the salamander's mass?

SOLUTION

Given: m_c = mass of carp = 5.00 kg

$\mathbf{v_{s,i}}$ = initial velocity of salamander = 3.60 m/s to the right

$\mathbf{v_{c,i}}$ = initial velocity of carp = 2.20 m/s to the right

$\mathbf{v_f}$ = final velocity = 3.50 m/s to the right

Unknown: m_s = mass of salamander = ?

Use the equation for a perfectly inelastic collision and rearrange it to solve for m_s.

$$m_s\mathbf{v_{s,i}} + m_c\mathbf{v_{c,i}} = (m_s + m_c)\mathbf{v_f}$$

$$m_s = \frac{m_c\mathbf{v_f} - m_c\mathbf{v_{c,i}}}{\mathbf{v_{s,i}} - \mathbf{v_f}}$$

$$m_s = \frac{(5.00\text{ kg})(3.50\text{ m/s}) - (5.00\text{ kg})(2.20\text{ m/s})}{3.60\text{ m/s} - 3.50\text{ m/s}}$$

$$m_s = \frac{17.5\text{ kg}\bullet\text{m/s} - 11.0\text{ kg}\bullet\text{m/s}}{0.10\text{ m/s}}$$

$$m_s = \frac{6.5\text{ kg}\bullet\text{m/s}}{0.10\text{ m/s}}$$

$$m_s = 65\text{ kg}$$

ADDITIONAL PRACTICE

1. Zorba, an English mastiff with a mass of 155 kg, jumps forward horizontally at a speed of 6.0 m/s into a boat that is floating at rest. After the jump, the boat and Zorba move with a velocity of 2.2 m/s forward. Calculate the boat's mass.
2. Yvonne van Gennip of the Netherlands ice skated 10.0 km with an average speed of 10.8 m/s. Suppose van Gennip crosses the finish line at her average speed and takes a huge bouquet of flowers handed to her by a fan. As a result, her speed drops to 10.01 m/s. If van Gennip's mass is 63.0 kg, what is the mass of the bouquet?

3. The world's largest guitar was built by a group of high school students in Indiana. Suppose that this guitar is placed on a light cart. The cart and guitar are then pushed with a velocity of 4.48 m/s to the right. One of the students tries to slow the cart by stepping on it as it passes by her. The new velocity of the cart, guitar, and student is 4.00 m/s to the right. If the student's mass is 54 kg, what is the mass of the guitar? Assume the mass of the cart is negligible.
4. The longest passenger buses in the world operate in Zaire. These buses are more than 30 m long, have two trailers, and have a total mass of 28×10^3 kg. Imagine a safety test involving one of these buses and a truck with a mass of 12×10^3 kg. The truck with an unknown velocity hits a bus that is at rest so that the two vehicles move forward together with a speed of 3.0 m/s. Calculate the truck's velocity prior to the collision.
5. Sumo wrestlers must be very heavy to be successful in their sport, which involves pushing the rival out of the ring. One of the greatest sumo champions, Akebono, had a mass of 227 kg. The heaviest sumo wrestler ever, Konishiki, at one point had a mass of 267 kg. Suppose these two wrestlers are opponents in a match. Akebono moves left with a speed of 4.0 m/s, while Konishiki moves toward Akebono with an unknown speed. After the wrestlers undergo an inelastic collision, both have a velocity of zero. From this information, calculate Konishiki's velocity before colliding with Akebono.
6. Louis Borsi, of London, built a drivable car that had a mass of 9.50 kg and could move as fast as 24.0 km/h. Suppose the inventor falls out of this car and the car proceeds driverless to the north at its maximum speed. The inventor's young daughter, who has a mass of 32.0 kg, "catches" the car by jumping northward from a nearby stairway. The velocity of the car and girl is 11.0 km/h to the north. What was the velocity of Borsi's daughter as she jumped in the car?
7. In 1990, Roger Hickey of California attained a speed of 89 km/h while standing on a skateboard. Suppose Hickey is riding horizontally at his stand-up speed when he catches up to another skateboarder, who is moving at 69 km/h in the same direction. If the second skateboarder steps sideways onto Hickey's skateboard, the two riders move forward with a new speed. Calculate this speed, assuming that both skateboarders have equal, but unknown, masses and that the mass of the skateboard is negligible.
8. The white shark is the largest carnivorous fish in the world. The mass of a white shark can be as great as 3.0×10^3 kg. In spite of (or perhaps because of) the mass and ferocity of the shark, it is prized by commercial and sports fishers alike. Suppose Joe, who is one of these fishers, goes to a cliff that overlooks the ocean. To see if the sharks are biting, Joe drops a 2.5×10^2 kg fish as bait into the ocean below. As it so happens, a 3.0×10^3 kg white shark is prowling the ocean floor just below the cliff. The shark sees the bait, which is sinking straight down at a speed of 3.0 m/s. The shark swims upward with

a speed of 1.0 m/s to swallow the bait. What is the velocity of the shark right after it has swallowed the bait?

9. The heaviest cow on record had a mass of 2.267×10^3 kg and lived in Maine at the beginning of the twentieth century. Imagine that during an agricultural exhibition, the cow's owner puts the cow on a railed cart that has a mass of 5.00×10^2 kg and pushes the cow and cart left to the stage with a speed of 2.00 m/s. Another farmer puts his cow, which has a mass of 1.800×10^3 kg, on an identical cart and pushes it toward the stage from the opposite direction with a speed of 1.40 m/s. The carts collide and stick together. What is the velocity of the cows after the collision?