Momentum and Collisions

ADDITIONAL PRACTICE G

Givens | *Solutions*

1. $m_2 = 0.500\ m_1$

$v_{1,i} = 3.680 \times 10^3$ km/h

$v_{1,f} = -4.40 \times 10^2$ km/h

$v_{2,f} = 5.740 \times 10^3$ km/h

Momentum conservation

$$m_1v_{1,i} + m_2v_{2,i} = m_1v_{1,f} + m_2v_{2,f}$$

$$v_{2,i} = \frac{m_1v_{1,f} + m_2v_{2,f} - m_1v_{1,i}}{m_2} = \frac{m_1v_{1,f} + (0.500)m_1v_{2,f} - m_1v_{1,i}}{(0.500)m_1}$$

$$v_{2,i} = (2.00)v_{1,f} + v_{2,f} - (2.00)v_{1,i} = (2.00)(-4.40 \times 10^2 \text{ km/h}) + 5.740 \times 10^3 \text{ km/h} - (2.00)(3.680 \times 10^3 \text{ km/h}) = -8.80 \times 10^2 \text{ km/h} + 5.740 \times 10^3 \text{ km/h} - 7.36 \times 10^3 \text{ km/h}$$

$$v_{2,i} = \boxed{-2.50 \times 10^3 \text{ km/h}}$$

Conservation of kinetic energy (check)

$$\tfrac{1}{2}m_1v_{1,i}^2 + \tfrac{1}{2}m_2v_{2,i}^2 = \tfrac{1}{2}m_1v_{1,f}^2 + \tfrac{1}{2}m_2v_{2,f}^2$$

$$\tfrac{1}{2}m_1v_{1,i}^2 + \tfrac{1}{2}(0.500)m_1v_{2,i}^2 = \tfrac{1}{2}m_1v_{1,f}^2 + \tfrac{1}{2}(0.500)m_1v_{2,f}^2$$

$$v_{1,i}^2 + (0.500)v_{2,i}^2 = v_{1,f}^2 + (0.500)v_{2,f}^2$$

$$(3.680 \times 10^3 \text{ km/h})^2 + (0.500)(-2.50 \times 10^3 \text{ km/h})^2 = (-4.40 \times 10^2 \text{ km/h})^2 + (0.500)(5.740 \times 10^3 \text{ km/h})^2$$

$$1.354 \times 10^7 \text{ km}^2/\text{h}^2 + 3.12 \times 10^6 \text{ km}^2/\text{h}^2 = 1.94 \times 10^5 \text{ km}^2/\text{h}^2 + 1.647 \times 10^7 \text{ km}^2/\text{h}^2$$

$$1.666 \times 10^7 \text{ km}^2/\text{h}^2 = 1.666 \times 10^7 \text{ km}^2/\text{h}^2$$

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2. $m_1 = 18.40$ kg

$m_2 = 56.20$ kg

$\mathbf{v_{2,i}} = 5.000$ m/s to the left $= -5.000$ m/s

$\mathbf{v_{2,f}} = 6.600 \times 10^{-2}$ m/s to the left $= -6.600 \times 10^{-2}$ m/s

$\mathbf{v_{1,f}} = 10.07$ m/s to the left $= -10.07$ m/s

Momentum conservation

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

$$\mathbf{v_{1,i}} = \frac{m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}} - m_2\mathbf{v_{2,i}}}{m_1}$$

$$\mathbf{v_{1,i}} = \frac{(18.40 \text{ kg})(-10.07 \text{ m/s}) + (56.20 \text{ kg})(-6.600 \times 10^{-2} \text{ m/s}) - (56.20 \text{ kg})(-5.000 \text{ m/s})}{18.40 \text{ kg}}$$

$$\mathbf{v_{1,i}} = \frac{-185.3 \text{ kg•m/s} - 3.709 \text{ kg•m/s} + 281.0 \text{ kg•m/s}}{18.40 \text{ kg}} = \frac{92.0 \text{ kg•m/s}}{18.40 \text{ kg}} = 5.00 \text{ m/s}$$

$$\mathbf{v_{1,i}} = \boxed{5.00 \text{ m/s to the right}}$$

Conservation of kinetic energy (check)

$$\tfrac{1}{2}m_1v_{1,i}^2 + \tfrac{1}{2}m_2v_{2,i}^2 = \tfrac{1}{2}m_1v_{1,f}^2 + \tfrac{1}{2}m_2v_{2,f}^2$$

$$\tfrac{1}{2}(18.40 \text{ kg})(5.00 \text{ m/s})^2 + \tfrac{1}{2}(56.20)(-5.000 \text{ m/s})^2 = \tfrac{1}{2}(18.40 \text{ kg})(-10.07 \text{ ms})^2 + \tfrac{1}{2}(56.20 \text{ kg})(-6.600 \times 10^{-2} \text{ m/s})^2$$

$$2.30 \times 10^2 \text{ J} + 702.5 \text{ J} = 932.9 \text{ J} + 0.1224 \text{ J}$$

$$932 \text{ J} = 933 \text{ J}$$

The slight difference arises from rounding.

Givens	*Solutions*
3. $m_1 = m_2$ $\mathbf{v_{1,i}}$ = 5.0 m/s to the right = +5.0 m/s $\mathbf{v_{1,f}}$ = 2.0 m/s to the left = −2.0 m/s $\mathbf{v_{2,f}}$ = 5.0 m/s to the right = +5.0 m/s	Momentum conservation $m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$ $\mathbf{v_{2,i}} = \frac{m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}} - m_1\mathbf{v_{1,i}}}{m_2} = \mathbf{v_{1,f}} + \mathbf{v_{2,f}} - \mathbf{v_{1,i}}$ $\mathbf{v_{2,i}} = -2.0\text{ m/s} + 5.0\text{ m/s} - 5.0\text{ m/s} = -2.0\text{ m/s}$ $\mathbf{v_{2,i}} = \boxed{\text{2.0 m/s to the left}}$ Conservation of kinetic energy (check) $\frac{1}{2}m_iv_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2 = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$ $v_{1,i}^2 + v_{2,i}^2 = v_{1,f}^2 + v_{2,f}^2$ $(5.0\text{ m/s})^2 + (-2.0\text{ m/s})^2 = (-2.0\text{ m/s})^2 + (5.0\text{ m/s})^2$ $25\text{ m}^2/\text{s}^2 + 4.0\text{ m}^2/\text{s}^2 = 4.0\text{ m}^2/\text{s}^2 + 25\text{ m}^2/\text{s}^2$ $29\text{ m}^2/\text{s}^2 = 29\text{ m}^2/\text{s}^2$
4. m_1 = 45.0 g $\mathbf{v_{1,i}}$ = 273 km/h to the right = +273 km/h $\mathbf{v_{2,i}}$ = 0 km/h $\mathbf{v_{1,f}}$ = 91 km/h to the left = −91 km/h $\mathbf{v_{2,f}}$ = 182 km/h to the right = +182 km/h	Momentum conservation $m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$ $m_2 = \frac{m_1\mathbf{v_{1,f}} - m_1\mathbf{v_{1,i}}}{\mathbf{v_{2,i}} - \mathbf{v_{2,f}}} = \frac{(45.0\text{ g})(-91\text{ km/h}) - (45.0\text{ g})(273\text{ km/h})}{0\text{ km/h} - 182\text{ km/h}}$ $m_2 = \frac{-4.1 \times 10^3\text{ g}\bullet\text{km/h} - 12.3 \times 10^3\text{ g}\bullet\text{km/h}}{-182\text{ km/h}} = \frac{-16.4 \times 10^3\text{ g}\bullet\text{km/h}}{-182\text{ km/h}}$ $m_2 = \boxed{90.1\text{ g}}$ Conservation of kinetic energy (check) $\frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2 = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}$ $\frac{1}{2}(45.0\text{ g})(273 \times 10^3\text{ m/h})^2(1\text{ h}/3600\text{ s})^2 + \frac{1}{2}(90.1\text{ g})(0\text{ m/s})^2$ $= \frac{1}{2}(45.0\text{ g})(-91 \times 10^3\text{ m/h})^2(1\text{ h}/3600\text{ s})^2 + \frac{1}{2}(90.1\text{ g})(182 \times 10^3\text{ m/h})^2(1\text{ h}/3600\text{ s})^2$ 129 J + 0 J = 14 J + 115 J 129 J = 129 J

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5. $\mathbf{v_{1,i}}$ = 185 km/h to the right = +185 km/h

$\mathbf{v_{2,i}}$ = 0 km/h

$\mathbf{v_{i,f}}$ = 80.0 km/h to the left = −80.0 km/h

$m_1 = 5.70 \times 10^{-2}$ kg

Solutions

Momentum conservation

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

$$\left(\frac{m_1}{m_2}\right)\mathbf{v_{1,i}} - \left(\frac{m_1}{m_2}\right)\mathbf{v_{1,f}} = \mathbf{v_{2,f}} - \mathbf{v_{2,i}}$$

$$\left(\frac{m_1}{m_2}\right)[185 \text{ km/h} - (-80.0 \text{ km/h})] = \mathbf{v_{2,f}} - 0 \text{ km/h}$$

$$\mathbf{v_{2,f}} = \left(\frac{m_1}{m_2}\right)(265 \text{ km/h}) \text{ to the right}$$

Conservation of kinetic energy

$$KE_i = \tfrac{1}{2}m_1 v_{1,i}^2 + \tfrac{1}{2}m_2 v_{2,i}^2 = \tfrac{1}{2}m_1 v_{1,i}^2$$

$$KE_f = \tfrac{1}{2}m_1 v_{1,f}^2 + \tfrac{1}{2}m_2 v_{2,f}^2$$

$$\tfrac{1}{2}m_1 v_{1,i}^2 = \tfrac{1}{2}m_1 v_{1,f}^2 + \tfrac{1}{2}m_2 v_{2,f}^2$$

$$\left(\frac{m_1}{m_2}\right)(v_{1,i})^2 = \left(\frac{m_1}{m_2}\right)(v_{1,f})^2 + v_{2,f}^2$$

$$\left(\frac{m_1}{m_2}\right)(185 \text{ km/h})^2 = \left(\frac{m_1}{m_2}\right)(-80.0 \text{ km/h})^2 + v_{2,f}^2$$

$$\sqrt{\left(\frac{m_1}{m_2}\right)(3.42 \times 10^4 \text{ km}^2/\text{h}^2) - \left(\frac{m_1}{m_2}\right)(6.40 \times 10^3 \text{ km}^2/\text{h}^2)} = v_{2,f}$$

$$v_{2,f} = \sqrt{\left(\frac{m_1}{m_2}\right)(2.78 \times 10^4 \text{ km}^2/\text{h}^2)} = \sqrt{\frac{m_1}{m_2}}\left(167 \text{ km/h}\right)$$

Equating the two results for $v_{2,f}$ yields the ratio of m_1 to m_2.

$$\left(\frac{m_1}{m_2}\right)(265 \text{ km/h}) = \sqrt{\frac{m_1}{m_2}}(167 \text{ km/h})$$

$$265 \text{ km/h} = \sqrt{\frac{m_2}{m_1}}(167 \text{ km/h})$$

$$\frac{m_2}{m_1} = \left(\frac{265 \text{ km/h}}{167 \text{ km/h}}\right)^2 = 2.52$$

$$m_2 = (2.52)\, m_1 = (2.52)(5.70 \times 10^{-2} \text{ kg})$$

$$m_2 = \boxed{0.144 \text{ kg}}$$

Givens | *Solutions*

6. $m_1 = 4.00 \times 10^5$ kg

$m_2 = 1.60 \times 10^5$ kg

$\mathbf{v_{1,i}} = 32.0$ km/h to the right

$\mathbf{v_{2,i}} = 36.0$ km/h to the right

$\mathbf{v_{1,f}} = 35.5$ km/h to the right

Momentum conservation

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

$$\mathbf{v_{2,f}} = \frac{m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} - m_1\mathbf{v_{1,f}}}{m_2}$$

$$\mathbf{v_{2,f}} = \frac{(4.00 \times 10^5 \text{ kg})(32.0 \text{ km/h}) + (1.60 \times 10^5 \text{ kg})(36.0 \text{ km/h}) - (4.00 \times 10^5 \text{ kg})(35.5 \text{ km/h})}{1.60 \times 10^5 \text{ kg}}$$

$$\mathbf{v_{2,f}} = \frac{1.28 \times 10^7 \text{ kg•km/h} + 5.76 \times 10^6 \text{ kg•km/h} - 1.42 \times 10^7 \text{ kg•km/h}}{1.60 \times 10^5 \text{ kg}}$$

$$\mathbf{v_{2,f}} = \frac{4.4 \times 10^6 \text{ kg•km/h}}{1.60 \times 10^5 \text{ kg}}$$

$$\mathbf{v_{2,f}} = \boxed{28 \text{ km/h to the right}}$$

Conservation of kinetic energy (check)

$$\tfrac{1}{2}m_1v_{1,i}^{\,2} + \tfrac{1}{2}m_2v_{2,i}^{\,2} = \tfrac{1}{2}m_1v_{1,f}^{\,2} + \tfrac{1}{2}m_2v_{2,f}^{\,2}$$

$$\tfrac{1}{2}(4.00 \times 10^5 \text{ kg})(32.0 \times 10^3 \text{ m/h})^2(1 \text{ h}/3600 \text{ s})^2 + \tfrac{1}{2}(1.60 \times 10^5 \text{ kg})(36.0 \times 10^3 \text{ m/h})^2 (1 \text{ h}/3600 \text{ s})^2 = \tfrac{1}{2}(4.00 \times 10^5 \text{ kg})(35.5 \times 10^3 \text{ m/h})^2(1 \text{ h}/3600 \text{ s})^2 + \tfrac{1}{2}(1.60 \times 10^5 \text{ kg}) (28 \times 10^3 \text{ m/h})^2(1 \text{ h}/3600 \text{ s})^2$$

$$1.58 \times 10^7 \text{ J} + 8.00 \times 10^6 \text{ J} = 1.94 \times 10^7 \text{ J} + 4.8 \times 10^6 \text{ J}$$

$$2.38 \times 10^7 \text{ J} = 2.42 \times 10^7 \text{ J}$$

The slight difference arises from rounding.

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7. $m_1 = 5.50 \times 10^5$ kg

$m_2 = 2.30 \times 10^5$ kg

$\mathbf{v_{1,i}} = 5.00$ m/s to the right $= +5.00$ m/s

$\mathbf{v_{2,i}} = 5.00$ m/s to the left $= -5.00$ m/s

$\mathbf{v_{2,f}} = 9.10$ m/s to the right $= +9.10$ m/s

Momentum conservation

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

$$\mathbf{v_{1,f}} = \frac{m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} - m_2\mathbf{v_{2,f}}}{m_1}$$

$$\mathbf{v_{1,f}} = \frac{(5.50 \times 10^5 \text{ kg})(5.00 \text{ m/s}) + (2.30 \times 10^5 \text{ kg})(-5.00 \text{ m/s}) - (9.10 \text{ m/s})}{5.50 \times 10^5 \text{ kg}}$$

$$\mathbf{v_{1,f}} = \frac{2.75 \times 10^6 \text{ kg•m/s} - 1.15 \times 10^6 \text{ kg•m/s} - 2.09 \times 10^6 \text{ kg•m/s}}{5.50 \times 10^5 \text{ kg}} = -0.89 \text{ m/s right}$$

$$\mathbf{v_{1,f}} = \boxed{0.89 \text{ m/s left}}$$

Conservation of kinetic energy (check)

$$\tfrac{1}{2}m_1v_{1,i}^{\,2} + \tfrac{1}{2}m_2v_{2,i}^{\,2} = \tfrac{1}{2}m_1v_{1,f}^{\,2} + \tfrac{1}{2}m_2v_{2,f}^{\,2}$$

$$\tfrac{1}{2}(5.50 \times 10^5 \text{ kg})(5.00 \text{ m/s})^2 + \tfrac{1}{2}(2.30 \times 10^5 \text{ kg})(-5.00 \text{ m/s})^2 = \tfrac{1}{2}(5.50 \times 10^5 \text{ kg}) (-0.89 \text{ m/s})^2 + \tfrac{1}{2}(2.30 \times 10^5 \text{ kg})(9.10 \times \text{m/s})^2$$

$$6.88 \times 10^6 \text{ J} + 2.88 \times 10^6 \text{ J} = 2.2 \times 10^5 \text{ J} + 9.52 \times 10^6 \text{ J}$$

$$9.76 \times 10^6 \text{ J} = 9.74 \times 10^6 \text{ J}$$

The slight difference arises from rounding.

Momentum and Collisions

Problem G

ELASTIC COLLISIONS

PROBLEM

American juggler Bruce Sarafian juggled 11 identical balls at one time in 1992. Each ball had a mass of 0.20 kg. Suppose two balls have an elastic head-on collision during the act. The first ball moves away from the collision with a velocity of 3.0 m/s to the right, and the second ball moves away with a velocity of 4.0 m/s to the left. If the first ball's velocity before the collision is 4.0 m/s to the left, what is the velocity of the second ball before the collision?

SOLUTION

1. **DEFINE**

Given: $m_1 = m_2 = 0.20$ kg

$\mathbf{v_{1,i}}$ = initial velocity of ball 1 = 4.0 m/s to the left = –4.0 m/s to the right

$\mathbf{v_{1,f}}$ = final velocity of ball 1 = 3.0 m/s to the right

$\mathbf{v_{2,f}}$ = final velocity of ball 2 = 4.0 m/s to the left = –4.0 m/s to the right

Unknown: $\mathbf{v_{2,I}}$ = initial velocity of ball 2 = ?

2. **PLAN** **Choose the equation(s) or situation:** Use the equation for the conservation of momentum to determine the initial velocity of ball 2. Because both balls have identical masses, the mass terms cancel.

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

$$\mathbf{v_{1,i}} + \mathbf{v_{2,i}} = \mathbf{v_{1,f}} + \mathbf{v_{2,f}}$$

Rearrange the equation(s) to isolate the unknown(s):

$$\mathbf{v_{2,i}} = \mathbf{v_{1,f}} + \mathbf{v_{2,f}} - \mathbf{v_{1,i}}$$

3. **CALCULATE** **Substitute the values into the equation(s) and solve:**

$$\mathbf{v_{2,i}} = 3.0 \text{ m/s} - 4.0 \text{ m/s} - (-4.0 \text{ m/s})$$

$$\mathbf{v_{2,i}} = 3.0 \text{ m/s to the right}$$

4. **EVALUATE** Confirm your answer by making sure that kinetic energy is also conserved.

$$\frac{1}{2}m_1 v_{1,i}^{\ 2} + \frac{1}{2}m_2 v_{2,i}^{\ 2} = \frac{1}{2}m_1 v_{1,f}^{\ 2} + \frac{1}{2}m_2 v_{2,f}^{\ 2}$$

$$v_{1,i}^{\ 2} + v_{2,i}^{\ 2} = v_{1,f}^{\ 2} + v_{1,f}^{\ 2}$$

$$(-4.0 \text{ m/s})^2 + (3.0 \text{ m/s})^2 = (3.0 \text{ m/s})^2 + (-4.0 \text{ m/s})^2$$

$$16 \text{ m}^2/\text{s}^2 + 9.0 \text{ m}^2/\text{s}^2 = 9.0 \text{ m}^2/\text{s}^2 + 16 \text{ m}^2/\text{s}^2$$

$$25 \text{ m}^2/\text{s}^2 = 25 \text{ m}^2/\text{s}^2$$

ADDITIONAL PRACTICE

1. The moon's orbital speed around Earth is 3.680×10^3 km/h. Suppose the moon suffers a perfectly elastic collision with a comet whose mass is 50.0 percent that of the moon. (A partially inelastic collision would be a much more

realistic event.) After the collision, the moon moves with a speed of -4.40×10^2 km/h, while the comet moves away from the moon at -5.740×10^3 km/h. What is the comet's speed before the collision?

2. The largest beet root on record had a mass of 18.40 kg. The largest cabbage on record had a mass of 56.20 kg. Imagine these two vegetables traveling in opposite directions. The cabbage, which travels 5.000 m/s to the left, collides with the beet root. After the collision, the cabbage has a velocity of 6.600×10^{-2} m/s to the left, and the beet root has a velocity of 10.07 m/s to the left. What is the beet root's velocity before the perfectly elastic collision?

3. The first astronaut to walk in outer space without being tethered to a spaceship was Capt. Bruce McCandless. In 1984, he used a jet backpack, which cost about $15 million to design, to move freely about the exterior of the space shuttle *Challenger*. Imagine two astronauts working in outer space. Suppose they have equal masses and accidentally run into each other. The first astronaut moves 5.0 m/s to the right before the collision and 2.0 m/s to the left afterwards. If the second astronaut moves 5.0 m/s to the right after the perfectly elastic collision, what was the second astronaut's initial velocity?

4. Speeds as high as 273 km/h have been recorded for golf balls. Suppose a golf ball whose mass is 45.0 g is moving to the right at 273 km/h and strikes another ball that is at rest. If after the perfectly elastic collision the golf ball moves 91 km/h to the left and the other ball moves 182 km/h to the right, what is the mass of the second ball?

5. Jana Novotna of what is now the Czech Republic has the strongest serve among her fellow tennis players. In 1993, she sent the ball flying with a speed of 185 km/h. Suppose a tennis ball moving to the right at this speed hits a moveable target of unknown mass. After the one-dimensional, perfectly elastic collision, the tennis ball bounces to the left with a speed of 80.0 km/h. If the tennis ball's mass is 5.70×10^{-2} kg, what is the target's mass? (Hint: Use the conservation of kinetic energy to solve for the second unknown quantity.)

6. Recall the two colliding snow trains in item 5 of the previous section. Suppose now that the collision between the two trains is perfectly elastic instead of inelastic. The train with a mass of 4.00×10^5 kg and a velocity of 32.0 km/h to the right is struck from behind by a second train with a mass of 1.60×10^5 kg and a velocity of 36.0 km/h to the right. If the first train's velocity increases to 35.5 km/h to the right, what is the final velocity of the second train after the collision?

7. A dump truck used in Canada has a mass of 5.50×10^5 kg when loaded and 2.30×10^5 kg when empty. Suppose two such trucks, one loaded and one empty, crash into each other at a monster truck show. The trucks are supplied with special bumpers that make a collision almost perfectly elastic. If the trucks hit each other at equal speeds of 5.00 m/s and the less massive truck recoils to the right with a speed of 9.10 m/s, what is the velocity of the full truck after the collision?