Momentum and Collisions

ADDITIONAL PRACTICE D

Givens / *Solutions*

1. $m_1 = 3.3 \times 10^3$ kg

$\mathbf{v_{1,i}} = 0$ m/s

$\mathbf{v_{2,i}} = 0$ m/s

$\mathbf{v_{2,f}} = 2.5$ m/s to the right $= +2.5$ m/s

$\mathbf{v_{1,f}} = 0.050$ m/s to the left $= -0.050$ m/s

Because the initial momentum is zero, the final momentum is also zero, and so

$$m_2 = \frac{-m_1\mathbf{v_{1,f}}}{\mathbf{v_{2,f}}} = \frac{-(3.3 \times 10^3 \text{ kg})(-0.050 \text{ m/s})}{2.5 \text{ m/s}} = \boxed{66 \text{ kg}}$$

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2. $m_1 = 1.25 \times 10^3$ kg

$\mathbf{v_{1,i}} = 0$ m/s

$\mathbf{v_{2,i}} = 0$ m/s

$\mathbf{v_{2,f}} = 1.40$ m/s backward $= -1.40$ m/s

$\Delta t_1 = 4.0$ s

$\Delta\mathbf{x_1} = 24$ cm forward $= +24$ cm

Because the initial momentum is zero, the final momentum is also zero, and so

$$\mathbf{v_{1,f}} = \frac{\Delta\mathbf{x_1}}{\Delta t_1} = \frac{0.24 \text{ m}}{4.0 \text{ s}} = 0.060 \text{ m/s forward}$$

$$m_2 = \frac{-m_1\mathbf{v_{1,f}}}{\mathbf{v_{2,f}}} = \frac{-(1.25 \times 10^3 \text{ kg})(0.060 \text{ m/s})}{-1.40 \text{ m/s}} = \boxed{54 \text{ kg}}$$

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3. $m_1 = 114$ kg

$\mathbf{v_{2,f}} = 5.32$ m/s backward $= -5.32$ m/s

$\mathbf{v_{1,f}} = 3.41$ m/s forward $= +3.41$ m/s

$m_2 = 60.0$ kg

$$m_1\mathbf{v_i} + m_2\mathbf{v_i} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

$$\mathbf{v_i} = \frac{m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}}{m_1 + m_2} = \frac{(114 \text{ kg})(3.41 \text{ m/s}) + (60.0 \text{ kg})(-5.32 \text{ m/s})}{114 \text{ kg} + 60.0 \text{ kg}}$$

$$\mathbf{v_i} = \frac{389 \text{ kg}\bullet\text{m/s} - 319 \text{ kg}\bullet\text{m/s}}{174 \text{ kg}} = \frac{7.0 \times 10^1 \text{ kg}\bullet\text{m/s}}{174 \text{ kg}} = 0.40 \text{ m/s}$$

$$\mathbf{v_i} = \boxed{0.40 \text{ m/s forward}}$$

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4. $m_1 = 5.4$ kg

$\mathbf{v_{1,f}} = 7.4$ m/s forward $= +7.4$ m/s

$\mathbf{v_{2,f}} = 1.4$ m/s backward $= -1.4$ m/s

$m_2 = 50.0$ kg

$$m_1\mathbf{v_i} + m_2\mathbf{v_i} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

$$\mathbf{v_i} = \frac{m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}}{m_1 + m_2} = \frac{(5.4 \text{ kg})(7.4 \text{ m/s}) + (50.0 \text{ kg})(-1.4 \text{ m/s})}{5.4 \text{ kg} + 50.0 \text{ kg}}$$

$$\mathbf{v_i} = \frac{4.0 \times 10^1 \text{ kg}\bullet\text{m/s} - 7.0 \times 10^1 \text{ kg}\bullet\text{m/s}}{55.4 \text{ kg}} = \frac{-3.0 \times 10^1 \text{ kg}\bullet\text{m/s}}{55.4 \text{ kg}} = -0.54 \text{ m/s}$$

$$\mathbf{v_i} = \boxed{0.54 \text{ m/s backward}}$$

Givens	*Solutions*
5. $m_1 = 3.4 \times 10^2$ kg $\mathbf{v_{2,f}}$ = 9.0 km/h northwest = −9.0 km/h $\mathbf{v_{1,f}}$ = 28 km/h southeast = +28 km/h $m_2 = 2.6 \times 10^2$ kg	$m_1\mathbf{v_i} + m_2\mathbf{v_i} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$ $\mathbf{v_i} = \frac{m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}}{m_1 + m_2} = \frac{(3.4 \times 10^2 \text{ kg})(28 \text{ km/h}) + (2.6 \times 10^2 \text{ kg})(-9.0 \text{ km/h})}{3.4 \times 10^2 \text{ kg} + 2.6 \times 10^2 \text{ kg}}$ $\mathbf{v_i} = \frac{9.5 \times 10^3 \text{ kg•km/h} - 2.3 \times 10^3 \text{ kg•km/h}}{6.0 \times 10^2 \text{ kg}} = \frac{7.2 \times 10^3 \text{ kg•km/h}}{6.0 \times 10^2 \text{ kg}}$ $\mathbf{v_i} = \boxed{12 \text{ km/h to the southeast}}$
6. $m_i = 3.6$ kg $m_2 = 3.0$ kg $\mathbf{v_{1,i}} = 0$ m/s $\mathbf{v_{2,i}} = 0$ m/s $\mathbf{v_{2,f}}$ = 2.0 m/s to the left = −2.0 m/s	Because the initial momentum is zero, the final momentum must also equal zero. $m_i\mathbf{v_{1,f}} = -m_2\mathbf{v_{2,f}}$ $\mathbf{v_{1,f}} = \frac{-m_2\mathbf{v_{2,f}}}{m_1} = \frac{-(3.0 \text{ kg})(-2.0 \text{ m/s})}{3.6 \text{ kg}} = 1.7 \text{ m/s} = \boxed{1.7 \text{ m/s to the right}}$
7. $m_1 = 449$ kg $\mathbf{v_{1,i}} = 0$ m/s $\mathbf{v_{2,i}} = 0$ m/s $\mathbf{v_{2,f}}$ = 4.0 m/s backward = −4.0 m/s $m_2 = 60.0$ kg $\Delta t = 3.0$ s	Because the initial momentum is zero, the final momentum must also equal zero. $\mathbf{v_{1,f}} = \frac{-m_2\mathbf{v_{2,f}}}{m_1} = \frac{-(60.0 \text{ kg})(-4.0 \text{ m/s})}{449 \text{ kg}} = 0.53 \text{ m/s} = 0.53 \text{ m/s forward}$ $\Delta\mathbf{x} = \mathbf{v_{1,f}}\Delta t = (0.53 \text{ m/s})(3.0 \text{ s}) = \boxed{1.6 \text{ m forward}}$

Momentum and Collisions

Problem D

CONSERVATION OF MOMENTUM

PROBLEM

Kangaroos are good runners that can sustain a speed of 56 km/h (15.5 m/s). Suppose a kangaroo is sitting on a log that is floating in a lake. When the kangaroo gets scared, she jumps off the log with a velocity of 15 m/s toward the bank. The log moves with a velocity of 3.8 m/s away from the bank. If the mass of the log is 250 kg, what is the mass of the kangaroo?

SOLUTION

1. **DEFINE**

Given: $\mathbf{v}_{\mathbf{k,i}}$ = initial velocity of kangaroo = 0 m/s

$\mathbf{v}_{\mathbf{1,i}}$ = initial velocity of log = 0 m/s

$\mathbf{v}_{\mathbf{k,f}}$ = final velocity of kangaroo = 15 m/s towards bank = +15 m/s

$\mathbf{v}_{\mathbf{1,f}}$ = final velocity of log = 3.8 m/s away from bank = −3.8 m/s

m_l = mass of log = 250 kg

Unknown: m_k = mass of kangaroo = ?

2. **PLAN** **Choose the equation(s) or situation:** Because the momentum of the kangaroo-log system is conserved and therefore remains constant, the total initial momentum of the kangaroo and log will equal the total final momentum of the kangaroo and log.

$$m_k\mathbf{v}_{\mathbf{k,i}} + m_l\mathbf{v}_{\mathbf{1,i}} = m_k\mathbf{v}_{\mathbf{k,f}} + m_l\mathbf{v}_{\mathbf{1,f}}$$

The initial velocities of the kangaroo and log are zero, and therefore the initial momentum for each of them is zero. It thus follows that the total final momentum for the kangaroo and log must also equal zero. The momentum-conservation equation reduces to the following:

$$m_k\mathbf{v}_{\mathbf{k,f}} + m_l\mathbf{v}_{\mathbf{1,f}} = 0$$

Rearrange the equation(s) and isolate the unknown(s):

$$m_k = \frac{-m_l\mathbf{v}_{\mathbf{1,f}}}{\mathbf{v}_{\mathbf{k,f}}}$$

3. **CALCULATE** **Substitute the values into the equation(s) and solve:**

$$m_k = \frac{-(250\text{ kg})(-3.8\text{ m/s})}{15\text{ m/s}}$$

$$m_k = 63\text{ kg}$$

4. **EVALUATE** Because the log is about four times as massive as the kangaroo, its velocity is about one-fourth as large as the kangaroo's velocity.

ADDITIONAL PRACTICE

1. The largest single publication in the world is the 1112-volume set of *British Parliamentary Papers* for 1968 through 1972. The complete set has a mass of 3.3×10^3 kg. Suppose the entire publication is placed on a cart that can move without friction. The cart is at rest, and a librarian is sitting on top of it, just having loaded the last volume. The librarian jumps off the cart with a horizontal velocity relative to the floor of 2.5 m/s to the right. The cart begins to roll to the left at a speed of 0.05 m/s. Assuming the cart's mass is negligible, what is the librarian's mass?
2. The largest grand piano in the world is *really* grand. Built in London, it has a mass of 1.25×10^3 kg. Suppose a pianist finishes playing this piano and pushes herself from the piano so that she rolls backwards with a speed of 1.4 m/s. Meanwhile, the piano rolls forward so that in 4.0 s it travels 24 cm at constant velocity. Assuming the stool that the pianist is sitting on has a negligible mass, what is the pianist's mass?
3. With a mass of 114 kg, *Baby Bird* is the smallest monoplane ever flown. Suppose the *Baby Bird* and pilot are coasting along the runway when the pilot jumps horizontally to the runway behind the plane. The pilot's velocity upon leaving the plane is 5.32 m/s backward. After the pilot jumps from the plane, the plane coasts forward with a speed of 3.40 m/s. If the pilot's mass equals 60.0 kg, what is the velocity of the plane and pilot before the pilot jumps?
4. The September 14, 1987, issue of the *New York Times* had a mass of 5.4 kg. Suppose a skateboarder picks up a copy of this issue to have a look at the comic pages while rolling backward on the skateboard. Upon realizing that the *New York Times* doesn't have a "funnies" section, the skateboarder promptly throws the entire issue in a recycling container. The newspaper is thrown forward with a speed of 7.4 m/s. When the skater throws the newspaper away, he rolls backward at a speed of 1.4 m/s. If the combined mass of the skateboarder and skateboard is assumed to be 50.0 kg, what is the initial velocity of the skateboarder and newspaper?
5. The longest bicycle in the world was built in New Zealand in 1988. It is more than 20 m in length, has a mass of 3.4×10^2 kg, and can be ridden by four people at a time. Suppose four people are riding the bike southeast when they realize that the street turns and that the bike won't make it around the corner. All four riders jump off the bike at the same time and with the same velocity (9.0 km/h to the northwest, as measured relative to Earth). The bicycle continues to travel forward with a velocity of 28 km/h to the southeast. If the combined mass of the riders is 2.6×10^2 kg, what is the velocity of the bicycle and riders immediately before the riders' escape?

6. The largest frog ever found was discovered in Cameroon in 1989. The frog's mass was nearly 3.6 kg. Suppose this frog is placed on a skateboard with a mass of 3.0 kg. The frog jumps horizontally off the skateboard to the right, and the skateboard rolls freely in the opposite direction with a speed of 2.0 m/s relative to the ground. If the frog and skateboard are initially at rest, what is the initial horizontal velocity of the frog?
7. In 1994, a pumpkin with a mass of 449 kg was grown in Canada. Suppose you want to push a pumpkin with this mass along a smooth, horizontal ramp. You give the pumpkin a good push, only to find yourself sliding backwards at a speed of 4.0 m/s. How far will the pumpkin slide 3.0 s after the push? Assume your mass to be 60.0 kg.