

Work and Energy

ADDITIONAL PRACTICE E

Givens

1. $m = 118 \text{ kg}$
 $h_i = 5.00 \text{ m}$
 $g = 9.81 \text{ m/s}^2$
 $v_i = 0 \text{ m/s}$
 $KE_f = 4.61 \text{ kJ}$

Solutions

$$PE_i + KE_i = PE_f + KE_f$$

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + KE_f$$

$$mgh_f = mgh_i + \frac{1}{2}mv_i^2 - KE_f$$

$$h_f = h_i + \frac{v_i^2}{2g} - \frac{KE_f}{mg} = 5.00 \text{ m} + \frac{(0 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} - \frac{4.61 \times 10^3 \text{ J}}{(118 \text{ kg})(9.81 \text{ m/s}^2)}$$

$$h_f = 5.00 \text{ m} - 3.98 \text{ m} = \boxed{1.02 \text{ m above the ground}}$$

2. $v_f = 42.7 \text{ m/s}$
 $h_f = 50.0 \text{ m}$
 $v_i = 0 \text{ m/s}$
 $g = 9.81 \text{ m/s}^2$

$$PE_i + KE_i = PE_f + KE_f$$

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

$$h_i = h_f + \frac{v_f^2 - v_i^2}{2g} = 50.0 \text{ m} + \frac{(42.7 \text{ m/s})^2 - (0 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = 50.0 \text{ m} + 92.9 \text{ m}$$

$$h_i = \boxed{143 \text{ m}}$$

The mass of the nut is not needed for the calculation.

3. $h_i = 3150 \text{ m}$
 $v_f = 60.0 \text{ m/s}$
 $KE_i = 0 \text{ J}$
 $g = 9.81 \text{ m/s}^2$

$$PE_i + KE_i = PE_f + KE_f$$

$$mgh_i = mgh_f + \frac{1}{2}mv_f^2$$

$$h_f = h_i - \frac{v_f^2}{2g} = 3150 \text{ m} - \frac{(60.0 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = 3150 \text{ m} - 183 \text{ m}$$

$$h_f = \boxed{2970 \text{ m}}$$

4. $h_i = 1.20 \times 10^2 \text{ m}$
 $h_f = 30.0 \text{ m}$
 $m = 72.0 \text{ kg}$
 $g = 9.81 \text{ m/s}^2$
 $KE_i = 0 \text{ J}$

$$PE_i + KE_i = PE_f + KE_f$$

$$PE_i - PE_f = KE_f$$

$$KE_f = \Delta PE = mg(h_i - h_f)$$

$$KE_f = (72.0 \text{ kg})(9.81 \text{ m/s}^2)(1.20 \times 10^2 \text{ m} - 30.0 \text{ m}) = (72.0 \text{ kg})(9.81 \text{ m/s}^2)(9.0 \times 10^1 \text{ m})$$

$$KE_f = \boxed{6.4 \times 10^4 \text{ J}}$$

$$v_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{(2)(6.4 \times 10^4 \text{ J})}{72.0 \text{ kg}}}$$

$$v_f = \boxed{42 \text{ m/s}}$$

Work and Energy *continue*

Givens

5. $h_f = 250.0 \text{ m}$

$$\Delta ME = -2.55 \times 10^5 \text{ J}$$

$$m = 250.0 \text{ kg}$$

$$g = 9.81 \text{ m/s}^2$$

Solutions

$$\Delta ME = PE_f - KE_i = mgh_f - \frac{1}{2}mv_i^2$$

$$v_i = \sqrt{2gh_f - \frac{2\Delta ME}{m}} = \sqrt{(2)(9.81 \text{ m/s}^2)(250.0 \text{ m}) - \frac{(2)(-2.55 \times 10^5 \text{ J})}{250.0 \text{ kg}}}$$

$$v_i = \sqrt{4.90 \times 10^3 \text{ m}^2/\text{s}^2 + 2.04 \times 10^3 \text{ m}^2/\text{s}^2} = \sqrt{6.94 \times 10^3 \text{ m}^2/\text{s}^2}$$

$$v_i = 83.3 \text{ m/s} = \boxed{3.00 \times 10^2 \text{ km/h}}$$

6. $h_i = 12.3 \text{ km}$

$$m = 120.0 \text{ g}$$

$$g = 9.81 \text{ m/s}^2$$

$$KE_i = 0 \text{ J}$$

$$\Delta h = h_i - h_f = 3.2 \text{ km}$$

$$PE_i + KE_i = PE_f + KE_f$$

$$PE_i - PE_f = KE_f$$

$$KE_f = PE_i - PE_f = mgh_i - mgh_f = mg\Delta h$$

$$KE_f = mg\Delta h = (0.1200 \text{ kg})(9.81 \text{ m/s}^2)(3.2 \times 10^3 \text{ m}) = \boxed{3.8 \times 10^3 \text{ J}}$$

$$PE_f = mgh_f = mg(h_i - \Delta h) = (0.1200 \text{ kg})(9.81 \text{ m/s}^2)(12.3 \times 10^3 \text{ m} - 3.2 \times 10^3 \text{ m})$$

$$PE_f = (0.1200 \text{ kg})(9.81 \text{ m/s}^2)(9.1 \times 10^3 \text{ m}) = \boxed{1.1 \times 10^4 \text{ J}}$$

Alternatively,

$$PE_f = PE_i - KE_f = mgh_i - KE_f$$

$$PE_f = (0.1200 \text{ kg})(9.81 \text{ m/s}^2)(12.3 \times 10^3 \text{ m}) - 3.8 \times 10^3 \text{ J} = 1.45 \times 10^4 \text{ J} - 3.8 \times 10^3 \text{ J}$$

$$PE_f = \boxed{1.07 \times 10^4 \text{ J}}$$

7. $h = 68.6 \text{ m}$

$$v = 35.6 \text{ m/s}$$

$$g = 9.81 \text{ m/s}^2$$

$$PE_f = 0 \text{ J}$$

$$KE_i = 0 \text{ J}$$

$$ME_i = PE_i = mgh$$

$$ME_f = KE_f = \frac{1}{2}mv^2$$

$$\text{percent of energy dissipated} = \frac{(ME_i - ME_f)(100)}{ME_i} = \left(\frac{mgh - \frac{1}{2}mv^2}{mgh} \right) (100)$$

$$\text{percent of energy dissipated} = \left(\frac{gh - \frac{1}{2}v^2}{gh} \right) (100)$$

$$\text{percent of energy dissipated} = \left(\frac{(9.81 \text{ m/s}^2)(68.6 \text{ m}) - \frac{1}{2}(35.6 \text{ m/s})^2}{(9.81 \text{ m/s}^2)(68.6 \text{ m})} \right) (100)$$

$$\text{percent of energy dissipated} = \frac{(673 \text{ J} - 634 \text{ J})(100)}{673 \text{ J}} = \frac{(39 \text{ J})(100)}{673 \text{ J}} = \boxed{5.8 \text{ percent}}$$

Work and Energy

Problem E**CONSERVATION OF MECHANICAL ENERGY****PROBLEM**

The largest apple ever grown had a mass of about 1.47 kg. Suppose you hold such an apple in your hand. You accidentally drop the apple, then manage to catch it just before it hits the ground. If the speed of the apple at that moment is 5.42 m/s, what is the kinetic energy of the apple? From what height did you drop it?

SOLUTION**1. DEFINE**

Given: $m = 1.47 \text{ kg}$
 $g = 9.81 \text{ m/s}^2$
 $v = 5.42 \text{ m/s}$

Unknown: $KE = ?$ $h = ?$

2. PLAN **Choose the equation(s) or situation:** Use the equations for kinetic and gravitational potential energy.

$$KE = \frac{1}{2}mv^2$$

$$PE_g = mgh$$

The zero level for gravitational potential energy is the ground. Because the apple ends its fall at the zero level, the final gravitational potential energy is zero.

$$PE_{g,f} = 0$$

The initial gravitational potential energy is measured at the point from which the apple is released.

$$PE_{g,i} = mgh$$

The apple is initially at rest, so the initial kinetic energy is zero.

$$KE_i = 0$$

The final kinetic energy is therefore:

$$KE_f = \frac{1}{2}mv^2$$

3. CALCULATE **Substitute the values into the equation(s) and solve:**

$$PE_{g,i} = (1.47 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})h$$

$$KE_f = \frac{1}{2}(1.47 \text{ kg})\left(5.42 \frac{\text{m}}{\text{s}}\right)^2$$

Solving for KE yields the following result:

$$KE = KE_f = \frac{1}{2}(1.47 \text{ kg})\left(5.42 \frac{\text{m}}{\text{s}}\right)^2 = 21.6 \text{ J}$$

Now use the principle of conservation for mechanical energy and the calculated quantity for KE_f to evaluate h .

$$ME_i = ME_f$$

$$PE_i + KE_i = PE_f + KE_f$$

$$PE_i + 0 \text{ J} = 0 \text{ J} + 21.6 \text{ J}$$

$$mgh = 21.6 \text{ J}$$

$$h = \frac{21.6 \text{ J}}{(1.47 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)} = 1.50 \text{ m}$$

4. **EVALUATE** Note that the height of the apple can be determined without knowing the apple's mass. This is because the conservation equation reduces to the equation relating speed, acceleration, and displacement: $v^2 = 2gh$.

ADDITIONAL PRACTICE

1. The largest watermelon ever grown had a mass of 118 kg. Suppose this watermelon were exhibited on a platform 5.00 m above the ground. After the exhibition, the watermelon is allowed to slide along to the ground along a smooth ramp. How high above the ground is the watermelon at the moment its kinetic energy is 4.61 kJ?
2. One species of eucalyptus tree, *Eucalyptus regnens*, grows to heights similar to those attained by California redwoods. Suppose a bird sitting on top of one specimen of eucalyptus tree drops a nut. If the speed of the falling nut at the moment it is 50.0 m above the ground is 42.7 m/s, how tall is the tree? Do you need to know the mass of the nut to solve this problem? Disregard air resistance.
3. In 1989, Michel Menin of France walked on a tightrope suspended under a balloon nearly at an altitude of 3150 m above the ground. Suppose a coin falls from Menin's pocket during his walk. How high above the ground is the coin when its speed is 60.0 m/s?
4. In 1936, Col. Harry Froboess of Switzerland jumped into the ocean from the airship *Graf Hindenburg*, which was 1.20×10^2 m above the water's surface. Assuming Froboess had a mass of 72.0 kg, what was his kinetic energy at the moment he was 30.0 m from the water's surface? What was his speed at that moment? Neglect the air resistance.
5. Suppose a motorcyclist rides a certain high-speed motorcycle. He reaches top speed and then coasts up a hill. The maximum height reached by the motorcyclist is 250.0 m. If 2.55×10^5 J of kinetic energy is dissipated by friction, what was the initial speed of the motorcycle? The combined mass of the motorcycle and motorcyclist is 250.0 kg.

6. The deepest mine ever drilled has a depth of 12.3 km (by contrast, Mount Everest has height of 8.8 km). Suppose you drop a rock with a mass of 120.0 g down the shaft of this mine. What would the rock's kinetic energy be after falling 3.2 km? What would the potential energy associated with the rock be at that same moment? Assume no air resistance and a constant free-fall acceleration.
7. *Desperado*, a roller coaster built in Nevada, has a vertical drop of 68.6 m. The roller coaster is designed so that the speed of the cars at the end of this drop is 35.6 m/s. Assume the cars are at rest at the start of the drop. What percent of the initial mechanical energy is dissipated by friction?