

# Forces and the Laws of Motion

## Additional Practice B

### Givens

### Solutions

1.  $F_1 = 7.5 \times 10^4 \text{ N north}$

$F_2 = 9.5 \times 10^4 \text{ N at } 15.0^\circ$   
north of west

$\theta_1 = 90.0^\circ$

$\theta_2 = 180.0^\circ - 15.0^\circ = 165.0^\circ$

$$F_{x,net} = \Sigma F_x = F_1(\cos \theta_1) + F_2(\cos \theta_2)$$

$$F_{x,net} = (7.5 \times 10^4 \text{ N})(\cos 90.0^\circ) + (9.5 \times 10^4 \text{ N})(\cos 165.0^\circ)$$

$$F_{x,net} = -9.2 \times 10^4 \text{ N}$$

$$F_{y,net} = \Sigma F_y = F_1(\sin \theta_1) + F_2(\sin \theta_2)$$

$$F_{y,net} = (7.5 \times 10^4 \text{ N})(\sin 90.0^\circ) + (9.5 \times 10^4 \text{ N})(\sin 165.0^\circ)$$

$$F_{y,net} = 7.5 \times 10^4 \text{ N} + 2.5 \times 10^4 \text{ N} = 10.0 \times 10^4 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{F_{y,net}}{F_{x,net}}\right) = \tan^{-1}\left(\frac{10.0 \times 10^4 \text{ N}}{-9.2 \times 10^4}\right) = -47^\circ$$

$$\theta = \boxed{47^\circ \text{ north of west}}$$

2.  $F_1 = 6.00 \times 10^2 \text{ N north}$

$F_2 = 7.50 \times 10^2 \text{ N east}$

$F_3 = 6.75 \times 10^2 \text{ N at } 30.0^\circ$   
south of east

$\theta_1 = 90.0^\circ$

$\theta_2 = 0.00^\circ$

$\theta_3 = -30.0^\circ$

$$F_{x,net} = \Sigma F_x = F_1(\cos \theta_1) + F_2(\cos \theta_2) + F_3(\cos \theta_3) = (6.00 \times 10^2 \text{ N})(\cos 90.0^\circ) + (7.50 \times 10^2 \text{ N})(\cos 0.00^\circ) + (6.75 \times 10^2 \text{ N})(\cos(-30.0^\circ))$$

$$F_{x,net} = 7.50 \times 10^2 \text{ N} + 5.85 \times 10^2 \text{ N} = 13.35 \times 10^2 \text{ N}$$

$$F_{y,net} = \Sigma F_y = F_1(\sin \theta_1) + F_2(\sin \theta_2) + F_3(\sin \theta_3) = (6.00 \times 10^2 \text{ N})(\sin 90.0^\circ) + (7.50 \times 10^2 \text{ N})(\sin 0.00^\circ) + (6.75 \times 10^2 \text{ N})(\sin(-30.0^\circ))$$

$$F_{y,net} = 6.00 \times 10^2 \text{ N} + (-3.38 \times 10^2 \text{ N}) = 2.62 \times 10^2 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{F_{y,net}}{F_{x,net}}\right) = \tan^{-1}\left(\frac{2.62 \times 10^2 \text{ N}}{13.35 \times 10^2 \text{ N}}\right)$$

$$\theta = \boxed{11.1^\circ \text{ north of east}}$$

3.  $F_1 = 2280.0 \text{ N upward}$

$F_2 = 2250.0 \text{ N downward}$

$F_3 = 85.0 \text{ N west}$

$F_4 = 12.0 \text{ N east}$

$$F_{y,net} = \Sigma F_y = F_1 + F_2 = 2280.0 \text{ N} + (-2250.0 \text{ N}) = 30.0 \text{ N}$$

$$F_{x,net} = \Sigma F_x = F_3 + F_4 = -85.0 \text{ N} + 12.0 \text{ N} = -73.0 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{F_{y,net}}{F_{x,net}}\right) = \tan^{-1}\left(\frac{30.0 \text{ N}}{-73.0 \text{ N}}\right) = -22.3^\circ$$

$$\theta = \boxed{22.3^\circ \text{ up from west}}$$

4.  $F_1 = 6.0 \text{ N}$

$F_2 = 8.0 \text{ N}$

$$F_{max} = F_1 + F_2 = 6.0 \text{ N} + 8.0 \text{ N}$$

$$F_{max} = \boxed{14.0 \text{ N}}$$

$$F_{min} = F_2 - F_1 = 8.0 \text{ N} - 6.0 \text{ N}$$

$$F_{min} = \boxed{2.0 \text{ N}}$$

## Forces and the Laws of Motion *continued*

### Givens

5.  $F_1 = 3.0 \text{ N east}$   
 $F_2 = 4.0 \text{ N south}$

### Solutions

$$F_{x,net} = F_1 + F_3(\cos \theta) = 0$$

$$F_3(\cos \theta) = -F_1 = -3.0 \text{ N}$$

$$F_{y,net} = F_2 + F_3(\sin \theta) = 0$$

$$F_3(\sin \theta) = -F_2 = -(-4.0 \text{ N}) = 4.0 \text{ N}$$

$$F_3 = \sqrt{[F_3(\cos \theta)]^2 + [F_3(\sin \theta)]^2} = \sqrt{(-3.0 \text{ N})^2 + (4.0 \text{ N})^2} = \sqrt{9.0 \text{ N}^2 + 16 \text{ N}^2} = \sqrt{25 \text{ N}^2}$$

$$F_3 = \boxed{5.0 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{F_3(\sin \theta)}{F_3(\cos \theta)}\right) = \tan^{-1}\left(\frac{4.0 \text{ N}}{-3.0 \text{ N}}\right) = -53^\circ$$

$$\theta = \boxed{53^\circ \text{ north of west}}$$

6.  $F_1 = 4.00 \times 10^3 \text{ N east}$   
 $F_2 = 5.00 \times 10^3 \text{ N north}$   
 $F_3 = 7.00 \times 10^3 \text{ N west}$   
 $F_4 = 9.00 \times 10^3 \text{ N south}$

$$F_{x,net} = F_1 + F_3 = 4.00 \times 10^3 \text{ N} + (-7.00 \times 10^3 \text{ N}) = -3.00 \times 10^3 \text{ N}$$

$$F_{y,net} = F_2 + F_4 = 5.00 \times 10^3 \text{ N} + (-9.00 \times 10^3 \text{ N}) = -4.00 \times 10^3 \text{ N}$$

$$F_{net} = \sqrt{(F_{x,net})^2 + (F_{y,net})^2} = \sqrt{(-3.00 \times 10^3 \text{ N})^2 + (-4.00 \times 10^3 \text{ N})^2}$$

$$F_{net} = \sqrt{9.00 \times 10^6 \text{ N}^2 + 16.0 \times 10^6 \text{ N}^2} = \sqrt{25.0 \times 10^6 \text{ N}^2}$$

$$F_{net} = \boxed{5.00 \times 10^3 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{F_{y,net}}{F_{x,net}}\right) = \tan^{-1}\left(\frac{-4.00 \times 10^3 \text{ N}}{-3.00 \times 10^3 \text{ N}}\right)$$

$$\theta = \boxed{53.1^\circ \text{ south of west}}$$

7.  $F_1 = 15.0 \text{ N}$   
 $\theta = 55.0^\circ$

$$F_y = F(\sin \theta) = (15.0 \text{ N})(\sin 55.0^\circ)$$

$$F_y = \boxed{12.3 \text{ N}}$$

$$F_x = F(\cos \theta) = (15.0 \text{ N})(\cos 55.0^\circ)$$

$$F_x = \boxed{8.60 \text{ N}}$$

8.  $F = 76 \text{ N}$   
 $\theta = 40.0^\circ$

$$F_x = F(\cos \theta) = (76 \text{ N})(\cos 40.0^\circ)$$

$$F_x = \boxed{58 \text{ N}}$$

$$F_y = F(\sin \theta) = (76 \text{ N})(\sin 40.0^\circ)$$

$$F_y = \boxed{49 \text{ N}}$$

9.  $F_1 = 350 \text{ N}$   
 $\theta_1 = 58.0^\circ$   
 $F_2 = 410 \text{ N}$   
 $\theta_2 = 43^\circ$

$$F_{y,net} = F_1(\sin \theta_1) + F_2(\sin \theta_2) = (350 \text{ N})(\sin 58^\circ) + (410 \text{ N})(\sin 43)$$

$$F_{y,net} = 3.0 \times 10^2 \text{ N} + 2.8 \times 10^2 \text{ N}$$

$$F_{y,net} = \boxed{580 \text{ N}}$$

10.  $F_1 = 7.50 \times 10^2 \text{ N}$   
 $\theta_1 = 40.0^\circ$   
 $F_2 = 7.50 \times 10^2 \text{ N}$   
 $\theta_2 = -40.0^\circ$

$$F_{y,net} = F_g = F_1(\cos \theta_1) + F_2(\cos \theta_2)$$

$$F_{y,net} = (7.50 \times 10^2 \text{ N})(\cos 40.0^\circ) + (7.50 \times 10^2 \text{ N})[\cos(-40.0^\circ)]$$

$$F_g = 575 \text{ N} + 575 \text{ N} = \boxed{1.150 \times 10^3 \text{ N}}$$

## Forces and the Laws of Motion

**Problem B****DETERMINING NET FORCE****PROBLEM**

Two soccer players kick a ball at the same instant. One player kicks with a force of 65 N to the north, while the other player kicks with a force of 88 N to the east. In what direction does the ball travel?

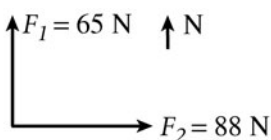
**SOLUTION****1. DEFINE**

**Given:**  $F_1 = 65 \text{ N north}$

$F_2 = 88 \text{ N east}$

**Unknown:**  $\theta = ?$

**Diagram**

**2. PLAN** Select a coordinate system and apply it to the free-body diagram.

Choose the positive  $x$ -axis to align with east and the positive  $y$ -axis to align with north.

**3. CALCULATE**

**Find the  $x$  and  $y$  components of all vectors.**

$$F_{1,x} = 0 \text{ N} \qquad F_{1,y} = 65 \text{ N}$$

$$F_{2,x} = 88 \text{ N} \qquad F_{2,y} = 0 \text{ N}$$

**Find the net external force in both the  $x$  and  $y$  directions.**

$$F_{x,net} = \Sigma F_x = F_{1,x} + F_{2,x} = 0 \text{ N} + 88 \text{ N} = 88 \text{ N}$$

$$F_{y,net} = \Sigma F_y = F_{1,y} + F_{2,y} = 65 \text{ N} + 0 \text{ N} = 65 \text{ N}$$

**Find the direction of the net external force.** Use the tangent function to find the angle  $\theta$  of  $F_{net}$ .

$$\theta = \tan^{-1} \left( \frac{F_{y,net}}{F_{x,net}} \right) = \tan^{-1} \left( \frac{65 \text{ N}}{88 \text{ N}} \right) = 36^\circ$$

$$\theta = \boxed{36^\circ \text{ north of east}}$$

**4. EVALUATE** The direction is about three-fourths of the way to the midpoint ( $45^\circ$ ) between north and east. This corresponds closely to the ratio of 65 N to 88 N (0.74).

**ADDITIONAL PRACTICE**

1. Two tugboats pull a barge across the harbor. One boat exerts a force of  $7.5 \times 10^4$  N north, while the second boat exerts a force of  $9.5 \times 10^4$  N at  $15.0^\circ$  north of west. Precisely, in what direction does the barge move?
2. Three workers move a car by pulling on three ropes. The first worker exerts a force of  $6.00 \times 10^2$  N to the north, the second a force of  $7.50 \times 10^2$  N to the east, and the third  $6.75 \times 10^2$  N at  $30.0^\circ$  south of east. In what precise direction does the car move?
3. Four forces are acting on a hot-air balloon:  $\mathbf{F}_1 = 2280.0$  N up,  $\mathbf{F}_2 = 2250.0$  N down,  $\mathbf{F}_3 = 85.0$  N west, and  $\mathbf{F}_4 = 12.0$  N east. What is the precise direction of the net external force on the balloon?
4. What is the magnitude of the largest net force that can be produced by combining a force of 6.0 N and a force of 8.0 N? What is the magnitude of the smallest such force?
5. Two friends grab different sides of a videotape cartridge and pull with forces of 3.0 N to the east and 4.0 N to the south, respectively. What force would a third friend need to exert on the cartridge in order to balance the other two forces? What would be that force's precise direction?
6. A four-way tug-of-war has four ropes attached to a metal ring. The forces on the ring are as follows:  $\mathbf{F}_1 = 4.00 \times 10^3$  N east,  $\mathbf{F}_2 = 5.00 \times 10^3$  N north,  $\mathbf{F}_3 = 7.00 \times 10^3$  N west, and  $\mathbf{F}_4 = 9.00 \times 10^3$  N south. What is the net force on the ring? What would be that force's precise direction?
7. A child pulls a toy by exerting a force of 15.0 N on a string that makes an angle of  $55.0^\circ$  with respect to the floor. What are the vertical and horizontal components of the force?
8. A shopper pushes a grocery cart by exerting a force on the handle. If the force equals 76 N at an angle of  $40.0^\circ$  below the horizontal, how much force is pushing the cart in the forward direction? What is the component of force pushing the cart against the floor?
9. Two paramedics are carrying a person on a stretcher. One paramedic exerts a force of 350 N at  $58^\circ$  above the horizontal and the other paramedic exerts a force of 410 N at  $43^\circ$  above the horizontal. What is the magnitude of the net upward force exerted by the paramedics?
10. A traffic signal is supported by two cables, each of which makes an angle of  $40.0^\circ$  with the vertical. If each cable can exert a maximum force of  $7.50 \times 10^2$  N, what is the largest weight they can support?