Name:_______________________ Class:________________ Date:________________

Two-Dimensional Motion and Vectors

Problem A - 3.2 Practice Problems

FINDING RESULTANT MAGNITUDE AND DIRECTION

PROBLEM

Cheetahs are, for short distances, the fastest land animals. In the course of a chase, cheetahs can also change direction very quickly. Suppose a cheetah runs straight north for 5.0 s, quickly turns, and runs 3.00×10^2 m west. If the magnitude of the cheetah's resultant displacement is 3.35×10^2 m, what is the cheetah's displacement and velocity during the first part of its run?

★Only do the circled problems.

SOLUTION

1. DEFINE

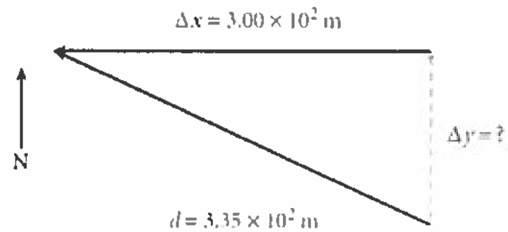
Given: $\Delta t_1 = 5.0$ s

$\Delta x = 3.00 \times 10^2$ m

$d = 3.35 \times 10^2$ m

Unknown: $\Delta y = ?$ $v_y = ?$

Diagram:



2. PLAN **Choose the equation(s) or situation:** Use the Pythagorean theorem to subtract one of the displacements at right angles from the total displacement, and thus determine the unknown component of displacement.

$$d^2 = \Delta x^2 + \Delta y^2$$

Use the equation relating displacement to constant velocity and time, and use the calculated value for Δy and the given value for Δt to solve for v.

$$\Delta v = \frac{\Delta y}{\Delta t}$$

Rearrange the equation(s) to isolate the unknown(s):

$$\Delta y^2 = d^2 - \Delta x^2$$

$$\Delta y = \sqrt{d^2 - \Delta x^2}$$

$$v_y = \frac{\Delta y}{\Delta t}$$

3. CALCULATE **Substitute the values into the equation(s) and solve:** Because the value for Δy is a displacement magnitude, only the positive root is used ($\Delta y > 0$).

$$\Delta y = \sqrt{(3.35 \times 10^2 \text{ m})^2 - (3.00 \times 10^2 \text{ m})^2}$$

$$= \sqrt{1.12 \times 10^5 \text{ m}^2 - 9.00 \times 10^4 \text{ m}^2}$$

$$= \sqrt{2.2 \times 10^4 \text{ m}}$$
$$= 1.5 \times 10^2 \text{ m, north}$$
$$v_y = \frac{1.5 \times 10^2 \text{ m}}{5.0 \text{ s}} = 3.0 \times 10^1 \text{ m/s, north}$$

4. EVALUATE The cheetah has a top speed of 30 m/s, or 107 km/h. This is equal to about 67 miles/h.

ADDITIONAL PRACTICE

1. An ostrich cannot fly, but it is able to run fast. Suppose an ostrich runs east for 7.95 s and then runs 161 m south, so that the magnitude of the ostrich's resultant displacement is 226 m. Calculate the magnitude of the ostrich's eastward component and its running speed.
2. The pronghorn antelope, found in North America, is the best long-distance runner among mammals. It has been observed to travel at an average speed of more than 55 km/h over a distance of 6.0 km. Suppose the antelope runs a distance of 5.0 km in a direction 11.5° north of east, turns, and then runs 1.0 km south. Calculate the resultant displacement.
3. Kangaroos can easily jump as far 8.0 m. If a kangaroo makes five such jumps westward, how many jumps must it make northward to have a northwest displacement with a magnitude of 68 m? What is the angle of the resultant displacement with respect to north?
4. In 1926, Gertrude Ederle of the United States became the first woman to swim across the English channel. Suppose Ederle swam 25.2 km east from the coast near Dover, England, then made a 90° turn and traveled south for 21.3 km to a point east of Calais, France. What was Ederle's resultant displacement?
5. The emperor penguin is the best diver among birds: the record dive is 483 m. Suppose an emperor penguin dives vertically to a depth of 483 m and then swims horizontally a distance of 225 m. What angle would the vector of the resultant displacement make with the water's surface? What is the magnitude of the penguin's resultant displacement?
6. A killer whale can swim as fast as 15 m/s. Suppose a killer whale swims in one direction at this speed for 8.0 s, makes a 90° turn, and continues swimming in the new direction with the same speed as before. After a certain time interval, the magnitude of the resultant displacement is 180.0 m. Calculate the amount of time the whale swims after changing direction.
7. Woodcocks are the slowest birds: their average speed during courtship displays can be as low as 8.00 km/h. Suppose a woodcock flies east for 15.0 min. It then turns and flies north for 22.0 min. Calculate the magnitude of the resultant displacement and the angle between the resultant displacement and the woodcock's initial displacement.

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Problem B

RESOLVING VECTORS

PROBLEM

Certain iguanas have been observed to run as fast as 10.0 m/s. Suppose an iguana runs in a straight line at this speed for 5.00 s. The direction of motion makes an angle of 30.0° to the east of north. Find the value of the iguana's northward displacement.

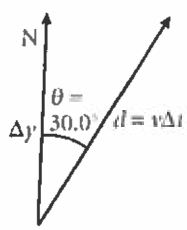
SOLUTION

1. DEFINE

Given: $v = 10.0$ m/s
$t = 5.00$ s
$\theta = 30.0°$

Unknown: $\Delta y = ?$

Diagram:



2. PLAN **Choose the equation (s) or situation:** The northern component of the vector is equal to the vector magnitude times the cosine of the angle between the vector and the northward direction.

$$\Delta y = d(\cos\theta)$$

Use the equation relating displacement with constant velocity and time, and substitute it for d in the previous equation.

$$d = v\Delta t$$
$$\Delta y = v\Delta t(\cos\theta)$$

3. CALCULATE **Substitute the values into the equation (s) and solve:**

$$\Delta y = \left(10.0\ \frac{\text{m}}{\text{s}}\right)(5.00\ \text{s})(\cos\ 30.0°)$$

$$= 43.3\ \text{m, north}$$

4. EVALUATE The northern component of the displacement vector is smaller than the displacement itself, as expected.

ADDITIONAL PRACTICE

1. A common flea can jump a distance of 33 cm. Suppose a flea makes five jumps of this length in the northwest direction. If the flea's northward displacement is 88 cm, what is the flea's westward displacement?

* Find both magnitude & direction

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2. The longest snake ever found was a python that was 10.0 m long. Suppose a coordinate system large enough to measure the python's length is drawn on the ground. The snake's tail is then placed at the origin and the snake's body is stretched so that it makes an angle of 60.0° with the positive x-axis. Find the x and y coordinates of the snake's head. (Hint: The y-coordinate is positive.)

3. A South-African sharp-nosed frog set a record for a triple jump by traveling a distance of 10.3 m. Suppose the frog starts from the origin of a coordinate system and lands at a point whose coordinate on the y-axis is equal to –6.10 m. What angle does the vector of displacement make with the negative y-axis? Calculate the x component of the frog.

4. The largest variety of grasshopper in the world is found in Malaysia. These grasshoppers can measure almost a foot (0.305 m) in length and can jump 4.5 m. Suppose one of these grasshoppers starts at the origin of a coordinate system and makes exactly eight jumps in a straight line that makes an angle of 35° with the positive x-axis. Find the grasshopper's displacements along the x- and y-axes. Assume both component displacements to be positive.

5. The landing speed of the space shuttle *Columbia* is 347 km/h. If the shuttle is landing at an angle of 15.0° with respect to the horizontal, what are the horizontal and the vertical components of its velocity?

6. In Virginia during 1994 Elmer Trett reached a speed of 372 km/h on his motorcycle. Suppose Trett rode northwest at this speed for 8.7 s. If the angle between east and the direction of Trett's ride was 60.0°, what was Trett's displacement east? What was his displacement north?

★ 1st solve for d using the velocity eq.

7. The longest delivery flight ever made by a twin-engine commercial jet took place in 1990. The plane covered a total distance of 14 890 km from Seattle, Washington to Nairobi, Kenya in 18.5 h. Assuming that the plane flew in a straight line between the two cities, find the magnitude of the average velocity of the plane. Also, find the eastward and southward components of the average velocity if the direction of the plane's flight was at an angle of 25.0° south of east.

8. The French bomber *Mirage IV* can fly over 2.3×10^3 km/h. Suppose this plane accelerates at a rate that allows it to increase its speed from 6.0×10^2 km/h to 2.3×10^3 km/h in a time interval of 120 s. If this acceleration is upward and at an angle of 35° with the horizontal, find the acceleration's horizontal and vertical components.

Problem C

ADDING VECTORS ALGEBRAICALLY

PROBLEM

The record for the longest nonstop closed-circuit flight by a model airplane was set in Italy in 1986. The plane flew a total distance of 1239 km. Assume that at some point the plane traveled 1.25×10^3 m to the east, then 1.25×10^3 m to the north, and finally 1.00×10^3 m to the southeast. Calculate the total displacement for this portion of the flight.

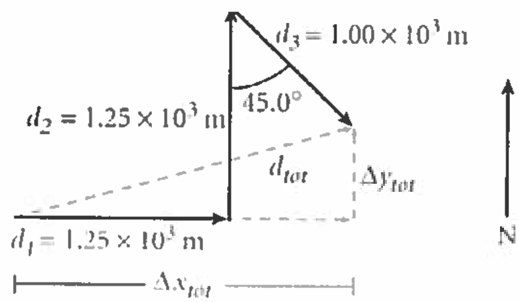
SOLUTION

1. DEFINE

Given: $d_1 = 1.25 \times 10^3$ m $d_2 = 1.25 \times 10^3$ m $d_3 = 1.00 \times 10^3$ m

Unknown: $\Delta x_{tot} = ?$ $\Delta y_{tot} = ?$ $d = ?$ $\theta = ?$

Diagram:



2. PLAN **Choose the equation(s) or situation:** Orient the displacements with respect to the x-axis of the coordinate system.

$\theta_1 = 0.00°$ $\theta_2 = 90.0°$ $\theta_3 = -45.0°$

Use this information to calculate the components of the total displacement along the x-axis and the y-axis.

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 + \Delta x_3$$
$$= d_1(\cos\theta_1) + d_2(\cos\theta_2) + d_3(\cos\theta_3)$$
$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 + \Delta y_3$$
$$= d_1(\sin\theta_1) + d_2(\sin\theta_2) + d_3(\sin\theta_3)$$

Use the components of the total displacement, the Pythagorean theorem, and the tangent function to calculate the total displacement.

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} \qquad \theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right)$$

3. CALCULATE **Substitute the values into the equation(s) and solve:**

$$\Delta x_{tot} = (1.25 \times 10^3 \text{ m})(\cos 0°) + (1.25 \times 10^3 \text{ m})(\cos 90.0°)$$
$$+ (1.00 \times 10^3 \text{ m})[\cos(-45.0°)]$$
$$= 1.25 \times 10^3 \text{ m} + 7.07 \times 10^2 \text{ m}$$
$$= 1.96 \times 10^3 \text{ m}$$
$$\Delta y_{tot} = (1.25 \times 10^3 \text{ m})(\sin 0°) + (1.25 \times 10^3 \text{ m})(\sin 90.0°)$$
$$+ (1.00 \times 10^3 \text{ m})[\sin(-45.0°)]$$
$$= 1.25 \times 10^3 \text{ m} + 7.07 \times 10^2 \text{ m}$$
$$= 0.543 \times 10^3 \text{ m}$$

$$d = \sqrt{(1.96 \times 10^3 \text{ m})^2 + (0.543 \times 10^3 \text{ m})^2}$$

$$d = \sqrt{3.84 \times 10^6 \text{ m}^2 + 2.95 \times 10^5 \text{ m}^2} = \sqrt{4.14 \times 10^6 \text{ m}^2}$$

$$d = 2.03 \times 10^3 \text{ m}$$

$$\theta = tan^{-1}\left(\frac{0.543 \times 10^3 \text{ m}}{1.96 \times 10^3 \text{ m}}\right)$$

$\theta = 15.5°$ north of east

4. **EVALUATE** The magnitude of the total displacement is slightly larger than that of the total displacement in the eastern direction alone.

ADDITIONAL PRACTICE

1. For six weeks in 1992, Akira Matsushima, from Japan, rode a unicycle more than 3000 mi across the United States. Suppose Matsushima is riding through a city. If he travels 250.0 m east on one street, then turns counterclockwise through a 120.0° angle and proceeds 125.0 m northwest along a diagonal street, what is his resultant displacement?
2. In 1976, the Lockheed SR-71A *Blackbird* set the record speed for any airplane: 3.53×10^3 km/h. Suppose you observe this plane ascending at this speed. For 20.0 s, it flies at an angle of 15.0° above the horizontal, then for another 10.0 s its angle of ascent is increased to 35.0°. Calculate the plane's total gain in altitude, its total horizontal displacement, and its resultant displacement.
3. Magnor Mydland of Norway constructed a motorcycle with a wheelbase of about 12 cm. The tiny vehicle could be ridden at a maximum speed of 11.6 km/h. Suppose this tiny motorcycle travels in the directions d_1 and d_2 where d_1 is 30° with the horizontal (upward and right) and d_2 is 45° with the vertical (down and to the right). Calculate d_1 and d_2, and determine how long it takes the motorcycle to reach a net displacement of 2.0×10^2 to the right.
4. The fastest propeller-driven aircraft is the Russian TU-95/142, which can reach a maximum speed of 925 km/h. For this speed, calculate the plane's resultant displacement if it travels east for 1.50 h, then turns 135° north-west and travels for 2.00 h.
5. In 1952, the ocean liner *United States* crossed the Atlantic Ocean in less than four days, setting the world record for commercial ocean-going vessels. The average speed for the trip was 57.2 km/h. Suppose the ship moves in a straight line eastward at this speed for 2.50 h. Then, due to a strong local current, the ship's course begins to deviate northward by 30.0°, and the ship follows the new course at the same speed for another 1.50 h. Find the resultant displacement for the 4.00 h period.